GLENCOE MATHEMATICS

Algebra 2

Chapter 5
Resource Masters

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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-828029-X
Skills Practice Workbook	0-07-828023-0
Practice Workbook	0-07-828024-9

ANSWERS FOR WORKBOOKS The answers for Chapter 5 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Algebra 2
Chapter 5 Resource Masters

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Contents

Teacher's Guide to Using the Chapter 5 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 5 Resource Masters* includes the core materials needed for Chapter 5. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 2 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 5-1. Encourage them to add these pages to their Algebra 2 Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in *Algebra 2* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 5 Resource Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 2. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 282–283. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

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5 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 5. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
binomial		
coefficient KOH·uh·FIH·shuhnt		
complex conjugates KAHN·jih·guht		
complex number		
degree		
extraneous solution ehk·STRAY·nee·uhs		
FOIL method		
imaginary unit		
like radical expressions		
like terms		

(continued on the next page)

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5 Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
monomial		
*n*th root		
polynomial		
power		
principal root		
pure imaginary number		
radical equation		
radical inequality		
rationalizing the denominator		
synthetic division sihn·THEH·tihk		
trinomial		

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5-1 Study Guide and Intervention

Monomials

Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables. Constants are monomials that contain no variables.

Negative Exponent	$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ for any real number $a \neq 0$ and any integer n.

When you **simplify an expression**, you rewrite it without parentheses or negative exponents. The following properties are useful when simplifying expressions.

Product of Powers	$a^m \cdot a^n = a^{m+n}$ for any real number a and integers m and n.
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$ for any real number $a \neq 0$ and integers m and n.
Properties of Powers	For a, b real numbers and m, n integers: $(a^m)^n = a^{mn}$ $(ab)^m = a^m b^m$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}$, $a \neq 0, b \neq 0$

Example **Simplify. Assume that no variable equals 0.**

a. $\mathbf{(3m^4n^{-2})(-5mn)^2}$

$$\begin{aligned}(3m^4n^{-2})(-5mn)^2 &= 3m^4n^{-2} \cdot 25m^2n^2\\ &= 75m^4m^2n^{-2}n^2\\ &= 75m^{4+2}n^{-2+2}\\ &= 75m^6\end{aligned}$$

b. $\mathbf{\frac{(-m^4)^3}{(2m^2)^{-2}}}$

$$\begin{aligned}\frac{(-m^4)^3}{(2m^2)^{-2}} &= \frac{-m^{12}}{\frac{1}{4m^4}}\\ &= -m^{12} \cdot 4m^4\\ &= -4m^{16}\end{aligned}$$

Exercises

Simplify. Assume that no variable equals 0.

1. $c^{12} \cdot c^{-4} \cdot c^6$

2. $\frac{b^8}{b^2}$

3. $(a^4)^5$

4. $\frac{x^{-2}y}{x^4y^{-1}}$

5. $\left(\frac{a^2b}{a^{-3}b^2}\right)^{-1}$

6. $\left(\frac{x^2y}{xy^3}\right)^2$

7. $\frac{1}{5}(-5a^2b^3)^2(abc)^2$

8. $m^7 \cdot m^8$

9. $\frac{8m^3n^2}{4mn^3}$

10. $\frac{2^3c^4t^2}{2^2c^4t^2}$

11. $4j(2j^{-2}k^2)(3j^3k^{-7})$

12. $\frac{2mn^2(3m^2n)^2}{12m^3n^4}$

Lesson 5-1

NAME ______________________________ DATE ____________ PERIOD _____

5-1 Study Guide and Intervention *(continued)*

Monomials

Scientific Notation

Scientific notation	A number expressed in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer

Example 1 **Express 46,000,000 in scientific notation.**

$46{,}000{,}000 = 4.6 \times 10{,}000{,}000$ $\quad 1 \leq 4.6 < 10$

$= 4.6 \times 10^7$ $\quad$ Write 10,000,000 as a power of ten.

Example 2 **Evaluate $\frac{3.5 \times 10^4}{5 \times 10^{-2}}$. Express the result in scientific notation.**

$$\frac{3.5 \times 10^4}{5 \times 10^{-2}} = \frac{3.5}{5} \times \frac{10^4}{10^{-2}}$$

$$= 0.7 \times 10^6$$

$$= 7 \times 10^5$$

Exercises

Express each number in scientific notation.

1. 24,300 **2.** 0.00099 **3.** 4,860,000

4. 525,000,000 **5.** 0.0000038 **6.** 221,000

7. 0.000000064 **8.** 16,750 **9.** 0.000369

Evaluate. Express the result in scientific notation.

10. $(3.6 \times 10^4)(5 \times 10^3)$ **11.** $(1.4 \times 10^{-8})(8 \times 10^{12})$ **12.** $(4.2 \times 10^{-3})(3 \times 10^{-2})$

13. $\frac{9.5 \times 10^7}{3.8 \times 10^{-2}}$ **14.** $\frac{1.62 \times 10^{-2}}{1.8 \times 10^5}$ **15.** $\frac{4.81 \times 10^8}{6.5 \times 10^4}$

16. $(3.2 \times 10^{-3})^2$ **17.** $(4.5 \times 10^7)^2$ **18.** $(6.8 \times 10^{-5})^2$

19. ASTRONOMY Pluto is 3,674.5 million miles from the sun. Write this number in scientific notation. **Source:** *New York Times Almanac*

20. CHEMISTRY The boiling point of the metal tungsten is 10,220°F. Write this temperature in scientific notation. **Source:** *New York Times Almanac*

21. BIOLOGY The human body contains 0.0004% iodine by weight. How many pounds of iodine are there in a 120-pound teenager? Express your answer in scientific notation. **Source:** *Universal Almanac*

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5-1 Skills Practice

Monomials

Simplify. Assume that no variable equals 0.

1. $b^4 \cdot b^3$

2. $c^5 \cdot c^2 \cdot c^2$

3. $a^{-4} \cdot a^{-3}$

4. $x^5 \cdot x^{-4} \cdot x$

5. $(g^4)^2$

6. $(3u)^3$

7. $(-x)^4$

8. $-5(2z)^3$

9. $-(-3d)^4$

10. $(-2t^2)^3$

11. $(-r^7)^3$

12. $\frac{s^{15}}{s^{12}}$

13. $\frac{k^9}{k^{10}}$

14. $(-3f^3g)^3$

15. $(2x)^2(4y)^2$

16. $-2gh(g^3h^5)$

17. $10x^2y^3(10xy^8)$

18. $\frac{24wz^7}{3w^3z^5}$

19. $\frac{-6a^4bc^8}{36a^7b^2c}$

20. $\frac{-10pq^4r}{-5p^3q^2r}$

Express each number in scientific notation.

21. 53,000

22. 0.000248

23. 410,100,000

24. 0.00000805

Evaluate. Express the result in scientific notation.

25. $(4 \times 10^3)(1.6 \times 10^{-6})$

26. $\frac{9.6 \times 10^7}{1.5 \times 10^{-3}}$

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5-1 Practice

Monomials

Simplify. Assume that no variable equals 0.

1. $n^5 \cdot n^2$

2. $y^7 \cdot y^3 \cdot y^2$

3. $t^9 \cdot t^{-8}$

4. $x^{-4} \cdot x^{-4} \cdot x^4$

5. $(2f^4)^6$

6. $(-2b^{-2}c^3)^3$

7. $(4d^2t^5v^{-4})(-5dt^{-3}v^{-1})$

8. $8u(2z)^3$

9. $\frac{12m^8y^6}{-9my^4}$

10. $\frac{-6s^5x^3}{18sx^7}$

11. $\frac{-27x^3(-x^7)}{16x^4}$

12. $\left(\frac{2}{3r^2s^3z^6}\right)^2$

13. $-(4w^{-3}z^{-5})(8w)^2$

14. $(m^4n^6)^4(m^3n^2p^5)^6$

15. $\left(\frac{3}{2}d^2f^4\right)^4\left(-\frac{4}{3}d^5f\right)^3$

16. $\left(\frac{2x^3y^2}{-x^2y^5}\right)^{-2}$

17. $\frac{(3x^{-2}y^3)(5xy^{-8})}{(x^{-3})^4y^{-2}}$

18. $\frac{-20(m^2v)(-v)^3}{5(-v)^2(-m^4)}$

Express each number in scientific notation.

19. 896,000

20. 0.000056

21. 433.7×10^8

Evaluate. Express the result in scientific notation.

22. $(4.8 \times 10^2)(6.9 \times 10^4)$

23. $(3.7 \times 10^9)(8.7 \times 10^2)$

24. $\frac{2.7 \times 10^6}{9 \times 10^{10}}$

25. COMPUTING The term *bit*, short for *binary digit*, was first used in 1946 by John Tukey. A single bit holds a zero or a one. Some computers use 32-bit numbers, or strings of 32 consecutive bits, to identify each address in their memories. Each 32-bit number corresponds to a number in our base-ten system. The largest 32-bit number is nearly 4,295,000,000. Write this number in scientific notation.

26. LIGHT When light passes through water, its velocity is reduced by 25%. If the speed of light in a vacuum is 1.86×10^5 miles per second, at what velocity does it travel through water? Write your answer in scientific notation.

27. TREES Deciduous and coniferous trees are hard to distinguish in a black-and-white photo. But because deciduous trees reflect infrared energy better than coniferous trees, the two types of trees are more distinguishable in an infrared photo. If an infrared wavelength measures about 8×10^{-7} meters and a blue wavelength measures about 4.5×10^{-7} meters, about how many times longer is the infrared wavelength than the blue wavelength?

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5-1 Reading to Learn Mathematics

Monomials

Pre-Activity **Why is scientific notation useful in economics?**

Read the introduction to Lesson 5-1 at the top of page 222 in your textbook.

Your textbook gives the U.S. public debt as an example from economics that involves large numbers that are difficult to work with when written in standard notation. Give an example from science that involves very large numbers and one that involves very small numbers.

Lesson 5-1

Reading the Lesson

1. Tell whether each expression is a monomial or not a monomial. If it is a monomial, tell whether it is a constant or not a constant.

a. $3x^2$

b. $y^2 + 5y - 6$

c. -73

d. $\frac{1}{z}$

2. Complete the following definitions of a negative exponent and a zero exponent.

For any real number $a \neq 0$ and any integer n, $a^{-n} =$ ____.

For any real number $a \neq 0$, $a^0 =$ ______.

3. Name the property or properties of exponents that you would use to simplify each expression. (Do not actually simplify.)

a. $\frac{m^8}{m^3}$

b. $y^6 \cdot y^9$

c. $(3r^2s)^4$

Helping You Remember

4. When writing a number in scientific notation, some students have trouble remembering when to use positive exponents and when to use negative ones. What is an easy way to remember this?

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5-1 Enrichment

Properties of Exponents

The rules about powers and exponents are usually given with letters such as *m, n,* and *k* to represent exponents. For example, one rule states that $a^m \cdot a^n = a^{m+n}$.

In practice, such exponents are handled as algebraic expressions and the rules of algebra apply.

Example 1 **Simplify $2a^2(a^{n+1} + a^{4n})$.**

$2a^2(a^{n+1} + a^{4n}) = 2a^2 \cdot a^{n+1} + 2a^2 \cdot a^{4n}$ Use the Distributive Law.

$= 2a^{2+n+1} + 2a^{2+4n}$ Recall $a^m \cdot a^n = a^{m+n}$.

$= 2a^{n+3} + 2a^{2+4n}$ Simplify the exponent $2 + n + 1$ as $n + 3$.

It is important always to collect *like* terms only.

Example 2 **Simplify $(a^n + b^m)^2$.**

$(a^n + b^m)^2 = (a^n + b^m)(a^n + b^m)$

F O I L

$= a^n \cdot a^n + a^n \cdot b^m + a^n \cdot b^m + b^m \cdot b^m$ The second and third terms are like terms.

$= a^{2n} + 2a^nb^m + b^{2m}$

Simplify each expression by performing the indicated operations.

1. $2^3 2^m$

2. $(a^3)^n$

3. $(4^n b^2)^k$

4. $(x^3 a^j)^m$

5. $(-ay^n)^3$

6. $(-b^k x)^2$

7. $(c^2)^{hk}$

8. $(-2d^n)^5$

9. $(a^2b)(a^nb^2)$

10. $(x^ny^m)(x^my^n)$

11. $\frac{a^n}{2}$

12. $\frac{12x^3}{4x^n}$

13. $(ab^2 - a^2b)(3a^n + 4b^n)$

14. $ab^2(2a^2b^{n-1} + 4ab^n + 6b^{n+1})$

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5-2

Study Guide and Intervention

Polynomials

Add and Subtract Polynomials

Polynomial	a monomial or a sum of monomials
Like Terms	terms that have the same variable(s) raised to the same power(s)

To add or subtract polynomials, perform the indicated operations and combine like terms.

Example 1 **Simplify $-6rs + 18r^2 - 5s^2 - 14r^2 + 8rs - 6s^2$.**

$-6rs + 18r^2 - 5s^2 - 14r^2 + 8rs - 6s^2$

$= (18r^2 - 14r^2) + (-6rs + 8rs) + (-5s^2 - 6s^2)$ Group like terms.

$= 4r^2 + 2rs - 11s^2$ Combine like terms.

Example 2 **Simplify $4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$.**

$4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$

$= 4xy^2 + 12xy - 7x^2y - 20xy - 5xy^2 + 8x^2y$ Distribute the minus sign.

$= (-7x^2y + 8x^2y) + (4xy^2 - 5xy^2) + (12xy - 20xy)$ Group like terms.

$= x^2y - xy^2 - 8xy$ Combine like terms.

Exercises

Simplify.

1. $(6x^2 - 3x + 2) - (4x^2 + x - 3)$

2. $(7y^2 + 12xy - 5x^2) + (6xy - 4y^2 - 3x^2)$

3. $(-4m^2 - 6m) - (6m + 4m^2)$

4. $27x^2 - 5y^2 + 12y^2 - 14x^2$

5. $(18p^2 + 11pq - 6q^2) - (15p^2 - 3pq + 4q^2)$

6. $17j^2 - 12k^2 + 3j^2 - 15j^2 + 14k^2$

7. $(8m^2 - 7n^2) - (n^2 - 12m^2)$

8. $14bc + 6b - 4c + 8b - 8c + 8bc$

9. $6r^2s + 11rs^2 + 3r^2s - 7rs^2 + 15r^2s - 9rs^2$

10. $-9xy + 11x^2 - 14y^2 - (6y^2 - 5xy - 3x^2)$

11. $(12xy - 8x + 3y) + (15x - 7y - 8xy)$

12. $10.8b^2 - 5.7b + 7.2 - (2.9b^2 - 4.6b - 3.1)$

13. $(3bc - 9b^2 - 6c^2) + (4c^2 - b^2 + 5bc)$

14. $11x^2 + 4y^2 + 6xy + 3y^2 - 5xy - 10x^2$

15. $\frac{1}{4}x^2 - \frac{3}{8}xy + \frac{1}{2}y^2 - \frac{1}{2}xy + \frac{1}{4}y^2 - \frac{3}{8}x^2$

16. $24p^3 - 15p^2 + 3p - 15p^3 + 13p^2 - 7p$

Lesson 5-2

5-2 Study Guide and Intervention *(continued)*

Polynomials

Multiply Polynomials You use the distributive property when you multiply polynomials. When multiplying binomials, the **FOIL** pattern is helpful.

FOIL Pattern	To multiply two binomials, add the products of F the *first* terms, O the *outer* terms, I the *inner* terms, and L the *last* terms.

Example 1 **Find $4y(6 - 2y + 5y^2)$.**

$4y(6 - 2y + 5y^2) = 4y(6) + 4y(-2y) + 4y(5y^2)$ Distributive Property

$= 24y - 8y^2 + 20y^3$ Multiply the monomials.

Example 2 **Find $(6x - 5)(2x + 1)$.**

$(6x - 5)(2x + 1) = 6x \cdot 2x + 6x \cdot 1 + (-5) \cdot 2x + (-5) \cdot 1$

First terms, Outer terms, Inner terms, Last terms

$= 12x^2 + 6x - 10x - 5$ Multiply monomials.

$= 12x^2 - 4x - 5$ Add like terms.

Exercises

Find each product.

1. $2x(3x^2 - 5)$

2. $7a(6 - 2a - a^2)$

3. $-5y^2(y^2 + 2y - 3)$

4. $(x - 2)(x + 7)$

5. $(5 - 4x)(3 - 2x)$

6. $(2x - 1)(3x + 5)$

7. $(4x + 3)(x + 8)$

8. $(7x - 2)(2x - 7)$

9. $(3x - 2)(x + 10)$

10. $3(2a + 5c) - 2(4a - 6c)$

11. $2(a - 6)(2a + 7)$

12. $2x(x + 5) - x^2(3 - x)$

13. $(3t^2 - 8)(t^2 + 5)$

14. $(2r + 7)^2$

15. $(c + 7)(c - 3)$

16. $(5a + 7)(5a - 7)$

17. $(3x^2 - 1)(2x^2 + 5x)$

18. $(x^2 - 2)(x^2 - 5)$

19. $(x + 1)(2x^2 - 3x + 1)$

20. $(2n^2 - 3)(n^2 + 5n - 1)$

21. $(x - 1)(x^2 - 3x + 4)$

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5-2 Skills Practice

Polynomials

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

1. $x^2 + 2x + 2$

2. $\frac{b^2c}{d^4}$

3. $8xz + \frac{1}{2}y$

Simplify.

4. $(g + 5) + (2g + 7)$

5. $(5d + 5) - (d + 1)$

6. $(x^2 - 3x - 3) + (2x^2 + 7x - 2)$

7. $(-2f^2 - 3f - 5) + (-2f^2 - 3f + 8)$

8. $(4r^2 - 6r + 2) - (-r^2 + 3r + 5)$

9. $(2x^2 - 3xy) - (3x^2 - 6xy - 4y^2)$

10. $(5t - 7) + (2t^2 + 3t + 12)$

11. $(u - 4) - (6 + 3u^2 - 4u)$

12. $-5(2c^2 - d^2)$

13. $x^2(2x + 9)$

14. $2q(3pq + 4q^4)$

15. $8w(hk^2 + 10h^3m^4 - 6k^5w^3)$

16. $m^2n^3(-4m^2n^2 - 2mnp - 7)$

17. $-3s^2y(-2s^4y^2 + 3sy^3 + 4)$

18. $(c + 2)(c + 8)$

19. $(z - 7)(z + 4)$

20. $(a - 5)^2$

21. $(2x - 3)(3x - 5)$

22. $(r - 2s)(r + 2s)$

23. $(3y + 4)(2y - 3)$

24. $(3 - 2b)(3 + 2b)$

25. $(3w + 1)^2$

Lesson 5-2

NAME ____ DATE ____________ PERIOD _____

5-2 Practice

Polynomials

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

1. $5x^3 + 2xy^4 + 6xy$

2. $-\frac{4}{3}ac - a^5d^3$

3. $\frac{12m^8n^9}{(m - n)^2}$

4. $25x^3z - x\sqrt{78}$

5. $6c^{-2} + c - 1$

6. $\frac{5}{r} + \frac{6}{s}$

Simplify.

7. $(3n^2 + 1) + (8n^2 - 8)$

8. $(6w - 11w^2) - (4 + 7w^2)$

9. $(-6n - 13n^2) + (-3n + 9n^2)$

10. $(8x^2 - 3x) - (4x^2 + 5x - 3)$

11. $(5m^2 - 2mp - 6p^2) - (-3m^2 + 5mp + p^2)$

12. $(2x^2 - xy + y^2) + (-3x^2 + 4xy + 3y^2)$

13. $(5t - 7) + (2t^2 + 3t + 12)$

14. $(u - 4) - (6 + 3u^2 - 4u)$

15. $-9(y^2 - 7w)$

16. $-9r^4y^2(-3ry^7 + 2r^3y^4 - 8r^{10})$

17. $-6a^2w(a^3w - aw^4)$

18. $5a^2w^3(a^2w^6 - 3a^4w^2 + 9aw^6)$

19. $2x^2(x^2 + xy - 2y^2)$

20. $-\frac{3}{5}ab^3d^2(-5ab^2d^5 - 5ab)$

21. $(v^2 - 6)(v^2 + 4)$

22. $(7a + 9y)(2a - y)$

23. $(y - 8)^2$

24. $(x^2 + 5y)^2$

25. $(5x + 4w)(5x - 4w)$

26. $(2n^4 - 3)(2n^4 + 3)$

27. $(w + 2s)(w^2 - 2ws + 4s^2)$

28. $(x + y)(x^2 - 3xy + 2y^2)$

29. BANKING Terry invests \$1500 in two mutual funds. The first year, one fund grows 3.8% and the other grows 6%. Write a polynomial to represent the amount Terry's \$1500 grows to in that year if x represents the amount he invested in the fund with the lesser growth rate.

30. GEOMETRY The area of the base of a rectangular box measures $2x^2 + 4x - 3$ square units. The height of the box measures x units. Find a polynomial expression for the volume of the box.

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5-2 Reading to Learn Mathematics

Polynomials

Pre-Activity **How can polynomials be applied to financial situations?**

Read the introduction to Lesson 5-2 at the top of page 229 in your textbook.

Suppose that Shenequa decides to enroll in a five-year engineering program rather than a four-year program. Using the model given in your textbook, how could she estimate the tuition for the fifth year of her program? (Do not actually calculate, but describe the calculation that would be necessary.)

Reading the Lesson

1. State whether the terms in each of the following pairs are *like terms* or *unlike terms*.

a. $3x^2, 3y^2$

b. $-m^4, 5m^4$

c. $8r^3, 8s^3$

d. $-6, 6$

2. State whether each of the following expressions is a *monomial*, *binomial*, *trinomial*, or *not a polynomial*. If the expression is a polynomial, give its degree.

a. $4r^4 - 2r + 1$

b. $\sqrt{3x}$

c. $5x + 4y$

d. $2ab + 4ab^2 - 6ab^3$

3. a. What is the FOIL method used for in algebra?

b. The FOIL method is an application of what property of real numbers?

c. In the FOIL method, what do the letters F, O, I, and L mean?

d. Suppose you want to use the FOIL method to multiply $(2x + 3)(4x + 1)$. Show the terms you would multiply, but do not actually multiply them.

F ________________

O ________________

I ________________

L ________________

Helping You Remember

4. You can remember the difference between *monomials*, *binomials*, and *trinomials* by thinking of common English words that begin with the same prefixes. Give two words unrelated to mathematics that start with *mono*-, two that begin with *bi*-, and two that begin with *tri*-.

Lesson 5-2

5-2 Enrichment

Polynomials with Fractional Coefficients

Polynomials may have fractional coefficients as long as there are no variables in the denominators. Computing with fractional coefficients is performed in the same way as computing with whole-number coefficients.

Simpliply. Write all coefficients as fractions.

1. $\left(\frac{3}{5}m - \frac{2}{7}p - \frac{1}{3}n\right) - \left(\frac{7}{3}p - \frac{5}{2}m - \frac{3}{4}n\right)$

2. $\left(\frac{3}{2}x - \frac{4}{3}y - \frac{5}{4}z\right) + \left(-\frac{1}{4}x + y + \frac{2}{5}z\right) + \left(-\frac{7}{8}x - \frac{6}{7}y + \frac{1}{2}z\right)$

3. $\left(\frac{1}{2}a^2 - \frac{1}{3}ab + \frac{1}{4}b^2\right) + \left(\frac{5}{6}a^2 + \frac{2}{3}ab - \frac{3}{4}b^2\right)$

4. $\left(\frac{1}{2}a^2 - \frac{1}{3}ab + \frac{1}{4}b^2\right) - \left(\frac{1}{3}a^2 - \frac{1}{2}ab + \frac{5}{6}b^2\right)$

5. $\left(\frac{1}{2}a^2 - \frac{1}{3}ab + \frac{1}{4}b^2\right) \cdot \left(\frac{1}{2}a - \frac{2}{3}b\right)$

6. $\left(\frac{2}{3}a^2 - \frac{1}{5}a + \frac{2}{7}\right) \cdot \left(\frac{2}{3}a^3 + \frac{1}{5}a^2 - \frac{2}{7}a\right)$

7. $\left(\frac{2}{3}x^2 - \frac{3}{4}x - 2\right) \cdot \left(\frac{4}{5}x - \frac{1}{6}x^2 - \frac{1}{2}\right)$

8. $\left(\frac{1}{6} + \frac{1}{3}x + \frac{1}{6}x^4 - \frac{1}{2}x^2\right) \cdot \left(\frac{1}{6}x^3 - \frac{1}{3} - \frac{1}{3}x\right)$

5-3 Study Guide and Intervention

Dividing Polynomials

Use Long Division To divide a polynomial by a monomial, use the properties of powers from Lesson 5-1.

To divide a polynomial by a polynomial, use a long division pattern. Remember that only like terms can be added or subtracted.

Example 1 **Simplify $\frac{12p^3t^2r - 21p^2qtr^2 - 9p^3tr}{3p^2tr}$.**

$$\frac{12p^3t^2r - 21p^2qtr^2 - 9p^3tr}{3p^2tr} = \frac{12p^3t^2r}{3p^2tr} - \frac{21p^2qtr^2}{3p^2tr} - \frac{9p^3tr}{3p^2tr}$$

$$= \frac{12}{3}p^{3-2}t^{2-1}r^{1-1} - \frac{21}{3}p^{2-2}qt^{1-1}r^{2-1} - \frac{9}{3}p^{3-2}t^{1-1}r^{1-1}$$

$$= 4pt - 7qr - 3p$$

Example 2 **Use long division to find $(x^3 - 8x^2 + 4x - 9) \div (x - 4)$.**

$$\begin{array}{r} x^2 - 4x - 12 \\ x - 4\overline{)x^3 - 8x^2 + 4x - 9} \\ \underline{(-)x^3 - 4x^2 } \\ -4x^2 + 4x \\ \underline{(-)-4x^2 + 16x} \\ -12x - 9 \\ \underline{(-)-12x + 48} \\ -57 \end{array}$$

The quotient is $x^2 - 4x - 12$, and the remainder is -57.

Therefore $\frac{x^3 - 8x^2 + 4x - 9}{x - 4} = x^2 - 4x - 12 - \frac{57}{x - 4}$.

Exercises

Simplify.

1. $\frac{18a^3 + 30a^2}{3a}$

2. $\frac{24mn^6 - 40m^2n^3}{4m^2n^3}$

3. $\frac{60a^2b^3 - 48b^4 + 84a^5b^2}{12ab^2}$

4. $(2x^2 - 5x - 3) \div (x - 3)$

5. $(m^2 - 3m - 7) \div (m + 2)$

6. $(p^3 - 6) \div (p - 1)$

7. $(t^3 - 6t^2 + 1) \div (t + 2)$

8. $(x^5 - 1) \div (x - 1)$

9. $(2x^3 - 5x^2 + 4x - 4) \div (x - 2)$

Lesson 5-3

5-3 Study Guide and Intervention *(continued)*

Dividing Polynomials

Use Synthetic Division

Synthetic division	a procedure to divide a polynomial by a binomial using coefficients of the dividend and the value of r in the divisor $x - r$

Use synthetic division to find $(2x^3 - 5x^2 + 5x - 2) \div (x - 1)$.

Step 1	Write the terms of the dividend so that the degrees of the terms are in descending order. Then write just the coefficients.	$2x^3 - 5x^2 + 5x - 2$ 2 −5 5 −2
Step 2	Write the constant r of the divisor $x - r$ to the left, In this case, $r = 1$. Bring down the first coefficient, 2, as shown.	1 | 2 −5 5 −2 ―――――― 2
Step 3	Multiply the first coefficient by r, $1 \cdot 2 = 2$. Write their product under the second coefficient. Then add the product and the second coefficient: $-5 + 2 = -3$.	1 | 2 −5 5 −2 2 ―――――― 2 −3
Step 4	Multiply the sum, −3, by r: $-3 \cdot 1 = -3$. Write the product under the next coefficient and add: $5 + (-3) = 2$.	1 | 2 −5 5 −2 2 −3 ―――――― 2 −3 2
Step 5	Multiply the sum, 2, by r: $2 \cdot 1 = 2$. Write the product under the next coefficient and add: $-2 + 2 = 0$. The remainder is 0.	1 | 2 −5 5 −2 2 −3 2 ―――――― 2 −3 2 0

Thus, $(2x^3 - 5x^2 + 5x - 2) \div (x - 1) = 2x^2 - 3x + 2$.

Exercises

Simplify.

1. $(3x^3 - 7x^2 + 9x - 14) \div (x - 2)$

2. $(5x^3 + 7x^2 - x - 3) \div (x + 1)$

3. $(2x^3 + 3x^2 - 10\text{x} - 3) \div (x + 3)$

4. $(x^3 - 8x^2 + 19x - 9) \div (x - 4)$

5. $(2x^3 + 10x^2 + 9x + 38) \div (x + 5)$

6. $(3x^3 - 8x^2 + 16x - 1) \div (x - 1)$

7. $(x^3 - 9x^2 + 17x - 1) \div (x - 2)$

8. $(4x^3 - 25x^2 + 4x + 20) \div (x - 6)$

9. $(6x^3 + 28x^2 - 7x + 9) \div (x + 5)$

10. $(x^4 - 4x^3 + x^2 + 7x - 2) \div (x - 2)$

11. $(12x^4 + 20x^3 - 24x^2 + 20x + 35) \div (3x + 5)$

NAME ____ DATE ____________ PERIOD _____

5-3 Skills Practice

Dividing Polynomials

Simplify.

1. $\frac{10c + 6}{2}$

2. $\frac{12x + 20}{4}$

3. $\frac{15y^3 + 6y^2 + 3y}{3y}$

4. $\frac{12x^2 - 4x - 8}{4x}$

5. $(15q^6 + 5q^2)(5q^4)^{-1}$

6. $(4f^5 - 6f^4 + 12f^3 - 8f^2)(4f^2)^{-1}$

7. $(6j^2k - 9jk^2) \div 3jk$

8. $(4a^2h^2 - 8a^3h + 3a^4) \div (2a^2)$

9. $(n^2 + 7n + 10) \div (n + 5)$

10. $(d^2 + 4d + 3) \div (d + 1)$

11. $(2s^2 + 13s + 15) \div (s + 5)$

12. $(6y^2 + y - 2)(2y - 1)^{-1}$

13. $(4g^2 - 9) \div (2g + 3)$

14. $(2x^2 - 5x - 4) \div (x - 3)$

15. $\frac{u^2 + 5u - 12}{u - 3}$

16. $\frac{2x^2 - 5x - 4}{x - 3}$

17. $(3v^2 - 7v - 10)(v - 4)^{-1}$

18. $(3t^4 + 4t^3 - 32t^2 - 5t - 20)(t + 4)^{-1}$

19. $\frac{y^3 - y^2 - 6}{y + 2}$

20. $\frac{2x^3 - x^2 - 19x + 15}{x - 3}$

21. $(4p^3 - 3p^2 + 2p) \div (p - 1)$

22. $(3c^4 + 6c^3 - 2c + 4)(c + 2)^{-1}$

23. GEOMETRY The area of a rectangle is $x^3 + 8x^2 + 13x - 12$ square units. The width of the rectangle is $x + 4$ units. What is the length of the rectangle?

Lesson 5-3

NAME ______________________________ DATE __________ PERIOD _____

5-3 Practice

Dividing Polynomials

Simplify.

1. $\frac{15r^{10} - 5r^8 + 40r^2}{5r^4}$

2. $\frac{6k^2m - 12k^3m^2 + 9m^3}{2km^2}$

3. $(-30x^3y + 12x^2y^2 - 18x^2y) \div (-6x^2y)$

4. $(-6w^3z^4 - 3w^2z^5 + 4w + 5z) \div (2w^2z)$

5. $(4a^3 - 8a^2 + a^2)(4a)^{-1}$

6. $(28d^3k^2 + d^2k^2 - 4dk^2)(4dk^2)^{-1}$

7. $\frac{f^2 + 7f + 10}{f + 2}$

8. $\frac{2x^2 + 3x - 14}{x - 2}$

9. $(a^3 - 64) \div (a - 4)$

10. $(b^3 + 27) \div (b + 3)$

11. $\frac{2x^3 + 6x + 152}{x + 4}$

12. $\frac{2x^3 + 4x - 6}{x + 3}$

13. $(3w^3 + 7w^2 - 4w + 3) \div (w + 3)$

14. $(6y^4 + 15y^3 - 28y - 6) \div (y + 2)$

15. $(x^4 - 3x^3 - 11x^2 + 3x + 10) \div (x - 5)$

16. $(3m^5 + m - 1) \div (m + 1)$

17. $(x^4 - 3x^3 + 5x - 6)(x + 2)^{-1}$

18. $(6y^2 - 5y - 15)(2y + 3)^{-1}$

19. $\frac{4x^2 - 2x + 6}{2x - 3}$

20. $\frac{6x^2 - x - 7}{3x + 1}$

21. $(2r^3 + 5r^2 - 2r - 15) \div (2r - 3)$

22. $(6t^3 + 5t^2 - 2t + 1) \div (3t + 1)$

23. $\frac{4p^4 - 17p^2 + 14p - 3}{2p - 3}$

24. $\frac{2h^4 - h^3 + h^2 + h - 3}{h^2 - 1}$

25. GEOMETRY The area of a rectangle is $2x^2 - 11x + 15$ square feet. The length of the rectangle is $2x - 5$ feet. What is the width of the rectangle?

26. GEOMETRY The area of a triangle is $15x^4 + 3x^3 + 4x^2 - x - 3$ square meters. The length of the base of the triangle is $6x^2 - 2$ meters. What is the height of the triangle?

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5-3 Reading to Learn Mathematics

Dividing Polynomials

Pre-Activity **How can you use division of polynomials in manufacturing?**

Read the introduction to Lesson 5-3 at the top of page 233 in your textbook.

Using the division symbol (÷), write the division problem that you would use to answer the question asked in the introduction. (Do not actually divide.)

Reading the Lesson

1. a. Explain in words how to divide a polynomial by a monomial.

b. If you divide a trinomial by a monomial and get a polynomial, what kind of polynomial will the quotient be?

2. Look at the following division example that uses the division algorithm for polynomials.

$$
\begin{array}{r|l}
 & 2x \quad + 4 \\
\hline
x - 4 & 2x^2 - 4x + 7 \\
 & \underline{2x^2 - 8x} \\
 & \qquad\ 4x + 7 \\
 & \qquad\ \underline{4x - 16} \\
 & \qquad\qquad\ 23
\end{array}
$$

Which of the following is the correct way to write the quotient?

A. $2x + 4$ **B.** $x - 4$ **C.** $2x + 4 + \frac{23}{x - 4}$ **D.** $\frac{23}{x - 4}$

3. If you use synthetic division to divide $x^3 + 3x^2 - 5x - 8$ by $x - 2$, the division will look like this:

$$
\begin{array}{r|rrrr}
2 & 1 & 3 & -5 & -8 \\
 & & 2 & 10 & 10 \\
\hline
 & 1 & 5 & 5 & \vert\ 2
\end{array}
$$

Which of the following is the answer for this division problem?

A. $x^2 + 5x + 5$ **B.** $x^2 + 5x + 5 + \frac{2}{x - 2}$

C. $x^3 + 5x^2 + 5x + \frac{2}{x - 2}$ **D.** $x^3 + 5x^2 + 5x + 2$

Helping You Remember

4. When you translate the numbers in the last row of a synthetic division into the quotient and remainder, what is an easy way to remember which exponents to use in writing the terms of the quotient?

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5-3 Enrichment

Oblique Asymptotes

The graph of $y = ax + b$, where $a \neq 0$, is called an oblique asymptote of $y = f(x)$ if the graph of f comes closer and closer to the line as $x \to \infty$ or $x \to -\infty$. ∞ is the mathematical symbol for **infinity**, which means *endless*.

For $f(x) = 3x + 4 + \frac{2}{x}$, $y = 3x + 4$ is an oblique asymptote because $f(x) - 3x - 4 = \frac{2}{x}$, and $\frac{2}{x} \to 0$ as $x \to \infty$ or $-\infty$. In other words, as $|x|$ increases, the value of $\frac{2}{x}$ gets smaller and smaller approaching 0.

Example **Find the oblique asymptote for $f(x) = \frac{x^2 + 8x + 15}{x + 2}$.**

$$\begin{array}{r|rrr} -2 & 1 & 8 & 15 \\ & & -2 & -12 \\ \hline & 1 & 6 & 3 \end{array}$$

Use synthetic division.

$$y = \frac{x^2 + 8x + 15}{x + 2} = x + 6 + \frac{3}{x + 2}$$

As $|x|$ increases, the value of $\frac{3}{x + 2}$ gets smaller. In other words, since $\frac{3}{x + 2} \to 0$ as $x \to \infty$ or $x \to -\infty$, $y = x + 6$ is an oblique asymptote.

Use synthetic division to find the oblique asymptote for each function.

1. $y = \frac{8x^2 - 4x + 11}{x + 5}$

2. $y = \frac{x^2 + 3x - 15}{x - 2}$

3. $y = \frac{x^2 - 2x - 18}{x - 3}$

4. $y = \frac{ax^2 + bx + c}{x - d}$

5. $y = \frac{ax^2 + bx + c}{x + d}$

5-4

Study Guide and Intervention

Factoring Polynomials

Factor Polynomials

Techniques for Factoring Polynomials	For any number of terms, check for: greatest common factor
	For two terms, check for: Difference of two squares $a^2 - b^2 = (a + b)(a - b)$ Sum of two cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ Difference of two cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
	For three terms, check for: Perfect square trinomials $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$ General trinomials $acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$
	For four terms, check for: Grouping $ax + bx + ay + by = x(a + b) + y(a + b)$ $= (a + b)(x + y)$

Example **Factor $24x^2 - 42x - 45$.**

First factor out the GCF to get $24x^2 - 42x - 45 = 3(8x^2 - 14x - 15)$. To find the coefficients of the x terms, you must find two numbers whose product is $8 \cdot (-15) = -120$ and whose sum is -14. The two coefficients must be -20 and 6. Rewrite the expression using $-20x$ and $6x$ and factor by grouping.

$8x^2 - 14x - 15 = 8x^2 - 20x + 6x - 15$ Group to find a GCF.

$= 4x(2x - 5) + 3(2x - 5)$ Factor the GCF of each binomial.

$= (4x + 3)(2x - 5)$ Distributive Property

Thus, $24x^2 - 42x - 45 = 3(4x + 3)(2x - 5)$.

Exercises

Factor completely. If the polynomial is not factorable, write *prime*.

1. $14x^2y^2 + 42xy^3$

2. $6mn + 18m - n - 3$

3. $2x^2 + 18x + 16$

4. $x^4 - 1$

5. $35x^3y^4 - 60x^4y$

6. $2r^3 + 250$

7. $100m^8 - 9$

8. $x^2 + x + 1$

9. $c^4 + c^3 - c^2 - c$

Lesson 5-4

NAME ____ DATE ____________ PERIOD _____

5-4 Study Guide and Intervention *(continued)*

Factoring Polynomials

Simplify Quotients In the last lesson you learned how to simplify the quotient of two polynomials by using long division or synthetic division. Some quotients can be simplified by using factoring.

Example **Simplify $\frac{8x^2 + 11x + 12}{2x^2 - 13x - 24}$.**

$\frac{8x^2 + 11x + 12}{2x^2 - 13x - 24} = \frac{(2x + 3)(x + 4)}{(x - 8)(2x + 3)}$ Factor the numerator and denominator.

$= \frac{x + 4}{x - 8}$ Divide. Assume $x \neq 8, -\frac{3}{2}$.

Exercises

Simplify. Assume that no denominator is equal to 0.

1. $\frac{x^2 - 7x + 12}{x^2 - x - 6}$

2. $\frac{x^2 + 6x + 5}{2x^2 - x - 3}$

3. $\frac{x^2 - 11x + 30}{x^2 - 5x - 6}$

4. $\frac{x^2 + x - 6}{x^2 - 7x + 10}$

5. $\frac{2x^2 + 5x - 3}{4x^2 + 11x - 3}$

6. $\frac{5x^2 + 9x - 2}{x^2 + 5x + 6}$

7. $\frac{4x^2 + 4x - 3}{2x^2 - x - 6}$

8. $\frac{6x^2 + 25x + 4}{x^2 + 6x + 8}$

9. $\frac{x^2 - 7x + 10}{3x^2 - 8x - 35}$

10. $\frac{4x^2 + 16x + 15}{2x^2 + x - 3}$

11. $\frac{3x^2 + 4x - 15}{2x^2 + 3x - 9}$

12. $\frac{x^2 - 14x + 49}{x^2 - 2x - 35}$

13. $\frac{x^2 - 81}{2x^2 - 23x + 45}$

14. $\frac{7x^2 + 11x - 6}{x^2 - 4}$

15. $\frac{4x^2 - 12x + 9}{2x^2 + 13x - 24}$

16. $\frac{4x^2 - 4x - 3}{8x^3 + 1}$

17. $\frac{y^3 - 64}{3y^2 - 17y + 20}$

18. $\frac{27x^3 - 8}{9x^2 - 4}$

NAME ______________________________ DATE ____________ PERIOD _____

5-4 Skills Practice

Factoring Polynomials

Factor completely. If the polynomial is not factorable, write *prime*.

1. $7x^2 - 14x$

2. $19x^3 - 38x^2$

3. $21x^3 - 18x^2y + 24xy^2$

4. $8j^3k - 4jk^3 - 7$

5. $a^2 + 7a - 18$

6. $2ak - 6a + k - 3$

7. $b^2 + 8b + 7$

8. $z^2 - 8z - 10$

9. $m^2 + 7m - 18$

10. $2x^2 - 3x - 5$

11. $4z^2 + 4z - 15$

12. $4p^2 + 4p - 24$

13. $3y^2 + 21y + 36$

14. $c^2 - 100$

15. $4f^2 - 64$

16. $d^2 - 12d + 36$

17. $9x^2 + 25$

18. $y^2 + 18y + 81$

19. $n^3 - 125$

20. $m^4 - 1$

Lesson 5-4

Simplify. Assume that no denominator is equal to 0.

21. $\frac{x^2 + 7x - 18}{x^2 + 4x - 45}$

22. $\frac{x^2 + 4x + 3}{x^2 + 6x + 9}$

23. $\frac{x^2 - 10x + 25}{x^2 - 5x}$

24. $\frac{x^2 + 6x - 7}{x^2 - 49}$

5-4 Practice

Factoring Polynomials

Factor completely. If the polynomial is not factorable, write *prime*.

1. $15a^2b - 10ab^2$

2. $3st^2 - 9s^3t + 6s^2t^2$

3. $3x^3y^2 - 2x^2y + 5xy$

4. $2x^3y - x^2y + 5xy^2 + xy^3$

5. $21 - 7t + 3r - rt$

6. $x^2 - xy + 2x - 2y$

7. $y^2 + 20y + 96$

8. $4ab + 2a + 6b + 3$

9. $6n^2 - 11n - 2$

10. $6x^2 + 7x - 3$

11. $x^2 - 8x - 8$

12. $6p^2 - 17p - 45$

13. $r^3 + 3r^2 - 54r$

14. $8a^2 + 2a - 6$

15. $c^2 - 49$

16. $x^3 + 8$

17. $16r^2 - 169$

18. $b^4 - 81$

19. $8m^3 - 25$

20. $2t^3 + 32t^2 + 128t$

21. $5y^5 + 135y^2$

22. $81x^4 - 16$

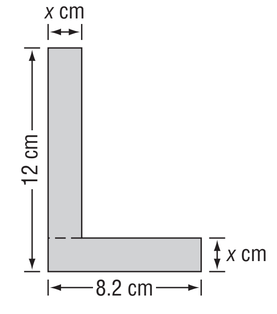
Simplify. Assume that no denominator is equal to 0.

23. $\dfrac{x^2 - 16}{x^2 + x - 20}$

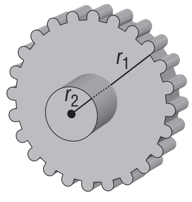
24. $\dfrac{x^2 - 16x + 64}{x^2 + x - 72}$

25. $\dfrac{3x^2 - 27}{x^3 - 27}$

26. DESIGN Bobbi Jo is using a software package to create a drawing of a cross section of a brace as shown at the right. Write a simplified, factored expression that represents the area of the cross section of the brace.



27. COMBUSTION ENGINES In an internal combustion engine, the up and down motion of the pistons is converted into the rotary motion of the crankshaft, which drives the flywheel. Let r_1 represent the radius of the flywheel at the right and let r_2 represent the radius of the crankshaft passing through it. If the formula for the area of a circle is $A = \pi r^2$, write a simplified, factored expression for the area of the cross section of the flywheel outside the crankshaft.



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5-4 Reading to Learn Mathematics

Factoring Polynomials

Pre-Activity **How does factoring apply to geometry?**

Read the introduction to Lesson 5-4 at the top of page 239 in your textbook.

If a trinomial that represents the area of a rectangle is factored into two binomials, what might the two binomials represent?

Reading the Lesson

1. Name three types of binomials that it is always possible to factor.

2. Name a type of trinomial that it is always possible to factor.

3. Complete: Since $x^2 + y^2$ cannot be factored, it is an example of a _____________ polynomial.

4. On an algebra quiz, Marlene needed to factor $2x^2 - 4x - 70$. She wrote the following answer: $(x + 5)(2x - 14)$. When she got her quiz back, Marlene found that she did not get full credit for her answer. She thought she should have gotten full credit because she checked her work by multiplication and showed that $(x + 5)(2x - 14) = 2x^2 - 4x - 70$.

a. If you were Marlene's teacher, how would you explain to her that her answer was not entirely correct?

b. What advice could Marlene's teacher give her to avoid making the same kind of error in factoring in the future?

Helping You Remember

5. Some students have trouble remembering the correct signs in the formulas for the sum and difference of two cubes. What is an easy way to remember the correct signs?

Lesson 5-4

NAME ______________________________ DATE ____________ PERIOD _____

5-4 Enrichment

Using Patterns to Factor

Study the patterns below for factoring the sum and the difference of cubes.

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

This pattern can be extended to other odd powers. Study these examples.

Example 1 **Factor $a^5 + b^5$.**

Extend the first pattern to obtain $a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$.

Check: $(a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4) = a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4$
$\qquad\qquad + a^4b - a^3b^2 + a^2b^3 - ab^4 + b^5$
$= a^5 \qquad\qquad\qquad + b^5$

Example 2 **Factor $a^5 - b^5$.**

Extend the second pattern to obtain $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$.

Check: $(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 + a^4b + a^3b^2 + a^2b^3 + ab^4$
$\qquad\qquad - a^4b - a^3b^2 - a^2b^3 - ab^4 - b^5$
$= a^5 \qquad\qquad\qquad - b^5$

In general, if n is an odd integer, when you factor $a^n + b^n$ or $a^n - b^n$, one factor will be either $(a + b)$ or $(a - b)$, depending on the sign of the original expression. The other factor will have the following properties:

- The first term will be a^{n-1} and the last term will be b^{n-1}.
- The exponents of a will decrease by 1 as you go from left to right.
- The exponents of b will increase by 1 as you go from left to right.
- The degree of each term will be $n - 1$.
- If the original expression was $a^n + b^n$, the terms will alternately have + and − signs.
- If the original expression was $a^n - b^n$, the terms will all have + signs.

Use the patterns above to factor each expression.

1. $a^7 + b^7$

2. $c^9 - d^9$

3. $e^{11} + f^{11}$

To factor $x^{10} - y^{10}$, change it to $(x^5 + y^5)(x^5 - y^5)$ and factor each binomial. Use this approach to factor each expression.

4. $x^{10} - y^{10}$

5. $a^{14} - b^{14}$

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5-5 Study Guide and Intervention

Roots of Real Numbers

Simplify Radicals

Square Root	For any real numbers a and b, if $a^2 = b$, then a is a square root of b.
nth Root	For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.
Real nth Roots of b, $\sqrt[n]{b}$, $-\sqrt[n]{b}$	1. If n is even and $b > 0$, then b has one positive root and one negative root. 2. If n is odd and $b > 0$, then b has one positive root. 3. If n is even and $b < 0$, then b has no real roots. 4. If n is odd and $b < 0$, then b has one negative root.

Example 1 **Simplify $\sqrt{49z^8}$.**

$\sqrt{49z^8} = \sqrt{(7z^4)^2} = 7z^4$

z^4 must be positive, so there is no need to take the absolute value.

Example 2 **Simplify $-\sqrt[3]{(2a-1)^6}$**

$-\sqrt[3]{(2a-1)^6} = -\sqrt[3]{[(2a-1)^2]^3} = (2a-1)^2$

Exercises

Simplify.

1. $\sqrt{81}$

2. $\sqrt[3]{-343}$

3. $\sqrt{144p^6}$

4. $\pm\sqrt{4a^{10}}$

5. $\sqrt[5]{243p^{10}}$

6. $-\sqrt[3]{m^6n^9}$

7. $\sqrt[3]{-b^{12}}$

8. $\sqrt{16a^{10}b^8}$

9. $\sqrt{121x^6}$

10. $\sqrt{(4k)^4}$

11. $\pm\sqrt{169r^4}$

12. $-\sqrt[3]{-27p^6}$

13. $-\sqrt{625y^2z^4}$

14. $\sqrt{36q^{34}}$

15. $\sqrt{100x^2y^4z^6}$

16. $\sqrt[3]{-0.027}$

17. $-\sqrt{-0.36}$

18. $\sqrt{0.64p^{10}}$

19. $\sqrt[4]{(2x)^8}$

20. $\sqrt{(11y^2)^4}$

21. $\sqrt[3]{(5a^2b)^6}$

22. $\sqrt{(3x-1)^2}$

23. $\sqrt[3]{(m-5)^6}$

24. $\sqrt{36x^2 - 12x + 1}$

Lesson 5-5

5-5 Study Guide and Intervention *(continued)*

Roots of Real Numbers

Approximate Radicals with a Calculator

Irrational Number	a number that cannot be expressed as a terminating or a repeating decimal

Radicals such as $\sqrt{2}$ and $\sqrt{3}$ are examples of irrational numbers. Decimal approximations for irrational numbers are often used in applications. These approximations can be easily found with a calculator.

Example **Approximate $\sqrt[5]{18.2}$ with a calculator.**

$\sqrt[5]{18.2} \approx 1.787$

Exercises

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{62}$

2. $\sqrt{1050}$

3. $\sqrt[3]{0.054}$

4. $-\sqrt[4]{5.45}$

5. $\sqrt{5280}$

6. $\sqrt{18,600}$

7. $\sqrt{0.095}$

8. $\sqrt[3]{-15}$

9. $\sqrt[5]{100}$

10. $\sqrt[6]{856}$

11. $\sqrt{3200}$

12. $\sqrt{0.05}$

13. $\sqrt{12,500}$

14. $\sqrt{0.60}$

15. $-\sqrt[4]{500}$

16. $\sqrt[3]{0.15}$

17. $\sqrt[6]{4200}$

18. $\sqrt{75}$

19. LAW ENFORCEMENT The formula $r = 2\sqrt{5L}$ is used by police to estimate the speed r in miles per hour of a car if the length L of the car's skid mark is measures in feet. Estimate to the nearest tenth of a mile per hour the speed of a car that leaves a skid mark 300 feet long.

20. SPACE TRAVEL The distance to the horizon d miles from a satellite orbiting h miles above Earth can be approximated by $d = \sqrt{8000h + h^2}$. What is the distance to the horizon if a satellite is orbiting 150 miles above Earth?

5-5

Skills Practice

Roots of Real Numbers

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{230}$

2. $\sqrt{38}$

3. $-\sqrt{152}$

4. $\sqrt{5.6}$

5. $\sqrt[3]{88}$

6. $\sqrt[3]{-222}$

7. $-\sqrt[4]{0.34}$

8. $\sqrt[5]{500}$

Simplify.

9. $\pm\sqrt{81}$

10. $\sqrt{144}$

11. $\sqrt{(-5)^2}$

12. $\sqrt{-5^2}$

13. $\sqrt{0.36}$

14. $-\sqrt{\frac{4}{9}}$

15. $\sqrt[3]{-8}$

16. $-\sqrt[3]{27}$

17. $\sqrt[3]{0.064}$

18. $\sqrt[5]{32}$

19. $\sqrt[4]{81}$

20. $\sqrt{y^2}$

21. $\sqrt[3]{125s^3}$

22. $\sqrt{64x^6}$

23. $\sqrt[3]{-27a^6}$

24. $\sqrt{m^8n^4}$

25. $-\sqrt{100p^4q^2}$

26. $\sqrt[4]{16w^4v^8}$

27. $\sqrt{(-3c)^4}$

28. $\sqrt{(a + b)^2}$

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5-5 Practice

Roots of Real Numbers

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{7.8}$

2. $-\sqrt{89}$

3. $\sqrt[3]{25}$

4. $\sqrt[3]{-4}$

5. $\sqrt[4]{1.1}$

6. $\sqrt[5]{-0.1}$

7. $\sqrt[6]{5555}$

8. $\sqrt[4]{(0.94)^2}$

Simplify.

9. $\sqrt{0.81}$

10. $-\sqrt{324}$

11. $-\sqrt[4]{256}$

12. $\sqrt[6]{64}$

13. $\sqrt[3]{-64}$

14. $\sqrt[3]{0.512}$

15. $\sqrt[5]{-243}$

16. $-\sqrt[4]{1296}$

17. $\sqrt[5]{\frac{-1024}{243}}$

18. $\sqrt[5]{243x^{10}}$

19. $\sqrt{(14a)^2}$

20. $\sqrt{-(14a)^2}$

21. $\sqrt{49m^2t^8}$

22. $\sqrt{\frac{16m^2}{25}}$

23. $\sqrt[3]{-64r^6w^{15}}$

24. $\sqrt{(2x)^8}$

25. $-\sqrt[4]{625s^8}$

26. $\sqrt[3]{216p^3q^9}$

27. $\sqrt{676x^4y^6}$

28. $\sqrt[3]{-27x^9y^{12}}$

29. $-\sqrt{144m^8n^6}$

30. $\sqrt[5]{-32x^5y^{10}}$

31. $\sqrt[6]{(m+4)^6}$

32. $\sqrt[3]{(2x+1)^3}$

33. $-\sqrt{49a^{10}b^{16}}$

34. $\sqrt[4]{(x-5)^8}$

35. $\sqrt[3]{343d^6}$

36. $\sqrt{x^2+10x+25}$

37. RADIANT TEMPERATURE Thermal sensors measure an object's *radiant* temperature, which is the amount of energy radiated by the object. The *internal* temperature of an object is called its *kinetic* temperature. The formula $T_r = T_k\sqrt[4]{e}$ relates an object's radiant temperature T_r to its kinetic temperature T_k. The variable e in the formula is a measure of how well the object radiates energy. If an object's kinetic temperature is 30°C and $e = 0.94$, what is the object's radiant temperature to the nearest tenth of a degree?

38. HERO'S FORMULA Salvatore is buying fertilizer for his triangular garden. He knows the lengths of all three sides, so he is using Hero's formula to find the area. Hero's formula states that the area of a triangle is $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the lengths of the sides of the triangle and s is half the perimeter of the triangle. If the lengths of the sides of Salvatore's garden are 15 feet, 17 feet, and 20 feet, what is the area of the garden? Round your answer to the nearest whole number.

5-5 Reading to Learn Mathematics

Roots of Real Numbers

Pre-Activity **How do square roots apply to oceanography?**

Read the introduction to Lesson 5-5 at the top of page 245 in your textbook.

Suppose the length of a wave is 5 feet. Explain how you would estimate the speed of the wave to the nearest tenth of a knot using a calculator. (Do not actually calculate the speed.)

Reading the Lesson

1. For each radical below, identify the radicand and the index.

a. $\sqrt[3]{23}$ radicand: ____________ index: ____________

b. $\sqrt{15x^2}$ radicand: ____________ index: ____________

c. $\sqrt[5]{-343}$ radicand: ____________ index: ____________

2. Complete the following table. (Do not actually find any of the indicated roots.)

Number	Number of Positive Square Roots	Number of Negative Square Roots	Number of Positive Cube Roots	Number of Negative Cube Roots
27				
−16				

3. State whether each of the following is *true* or *false*.

a. A negative number has no real fourth roots.

b. $\pm\sqrt{121}$ represents both square roots of 121.

c. When you take the fifth root of x^5, you must take the absolute value of x to identify the principal fifth root.

Helping You Remember

4. What is an easy way to remember that a negative number has no real square roots but has one real cube root?

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5-5 Enrichment

Approximating Square Roots

Consider the following expansion.

$$\left(a + \frac{b}{2a}\right)^2 = a^2 + \frac{2ab}{2a} + \frac{b^2}{4a^2}$$

$$= a^2 + b + \frac{b^2}{4a^2}$$

Think what happens if a is very great in comparison to b. The term $\frac{b^2}{4a^2}$ is very small and can be disregarded in an approximation.

$$\left(a + \frac{b}{2a}\right)^2 \approx a^2 + b$$

$$a + \frac{b}{2a} \approx \sqrt{a^2 + b}$$

Suppose a number can be expressed as $a^2 + b$, $a > b$. Then an approximate value of the square root is $a + \frac{b}{2a}$. You should also see that $a - \frac{b}{2a} \approx \sqrt{a^2 - b}$.

Example **Use the formula $\sqrt{a^2 \pm b} \approx a \pm \frac{b}{2a}$ to approximate $\sqrt{101}$ and $\sqrt{622}$.**

a. $\sqrt{101} = \sqrt{100 + 1} = \sqrt{10^2 + 1}$

Let $a = 10$ and $b = 1$.

$\sqrt{101} \approx 10 + \frac{1}{2(10)}$

≈ 10.05

b. $\sqrt{622} = \sqrt{625 - 3} = \sqrt{25^2 - 3}$

Let $a = 25$ and $b = 3$.

$\sqrt{622} \approx 25 - \frac{3}{2(25)}$

≈ 24.94

Use the formula to find an approximation for each square root to the nearest hundredth. Check your work with a calculator.

1. $\sqrt{626}$

2. $\sqrt{99}$

3. $\sqrt{402}$

4. $\sqrt{1604}$

5. $\sqrt{223}$

6. $\sqrt{80}$

7. $\sqrt{4890}$

8. $\sqrt{2505}$

9. $\sqrt{3575}$

10. $\sqrt{1{,}441{,}100}$

11. $\sqrt{290}$

12. $\sqrt{260}$

13. Show that $a - \frac{b}{2a} \approx \sqrt{a^2 - b}$ for $a > b$.

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5-6 Study Guide and Intervention

Radical Expressions

Lesson 5-6

Simplify Radical Expressions

Product Property of Radicals	For any real numbers *a* and *b*, and any integer $n > 1$: 1. if *n* is even and *a* and *b* are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$. 2. if *n* is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

To simplify a square root, follow these steps:

1. Factor the radicand into as many squares as possible.

2. Use the Product Property to isolate the perfect squares.

3. Simplify each radical.

Quotient Property of Radicals	For any real numbers *a* and $b \neq 0$, and any integer $n > 1$, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

To eliminate radicals from a denominator or fractions from a radicand, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

Example 1 **Simplify $\sqrt[3]{-16a^5b^7}$.**

$$\sqrt[3]{-16a^5b^7} = \sqrt[3]{(-2)^3 \cdot 2 \cdot a^3 \cdot a^2 \cdot (b^2)^3 \cdot b}$$
$$= -2ab^2\sqrt[3]{2a^2b}$$

Example 2 **Simplify $\sqrt{\frac{8x^3}{45y^5}}$.**

$$\sqrt{\frac{8x^3}{45y^5}} = \frac{\sqrt{8x^3}}{\sqrt{45y^5}}$$ Quotient Property

$$= \frac{\sqrt{(2x)^2 \cdot 2x}}{\sqrt{(3y^2)^2 \cdot 5y}}$$ Factor into squares.

$$= \frac{\sqrt{(2x)^2} \cdot \sqrt{2x}}{\sqrt{(3y^2)^2} \cdot \sqrt{5y}}$$ Product Property

$$= \frac{2|x|\sqrt{2x}}{3y^2\sqrt{5y}}$$ Simplify.

$$= \frac{2|x|\sqrt{2x}}{3y^2\sqrt{5y}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}}$$ Rationalize the denominator.

$$= \frac{2|x|\sqrt{10xy}}{15y^3}$$ Simplify.

Exercises

Simplify.

1. $5\sqrt{54}$

2. $\sqrt[4]{32a^9b^{20}}$

3. $\sqrt{75x^4y^7}$

4. $\sqrt{\frac{36}{125}}$

5. $\sqrt{\frac{a^6b^3}{98}}$

6. $\sqrt[3]{\frac{p^5q^3}{40}}$

5-6 Study Guide and Intervention *(continued)*

Radical Expressions

Operations with Radicals When you add expressions containing radicals, you can add only like terms or **like radical expressions**. Two radical expressions are called *like radical expressions* if both the indices and the radicands are alike.

To multiply radicals, use the Product and Quotient Properties. For products of the form $(a\sqrt{b} + c\sqrt{d}) \cdot (e\sqrt{f} + g\sqrt{h})$, use the FOIL method. To rationalize denominators, use **conjugates**. Numbers of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called conjugates. The product of conjugates is always a rational number.

Example 1 **Simplify $2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125}$.**

$2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125} = 2\sqrt{5^2 \cdot 2} + 4\sqrt{10^2 \cdot 5} - 6\sqrt{5^2 \cdot 5}$ Factor using squares.

$= 2 \cdot 5 \cdot \sqrt{2} + 4 \cdot 10 \cdot \sqrt{5} - 6 \cdot 5 \cdot \sqrt{5}$ Simplify square roots.

$= 10\sqrt{2} + 40\sqrt{5} - 30\sqrt{5}$ Multiply.

$= 10\sqrt{2} + 10\sqrt{5}$ Combine like radicals.

Example 2 **Simplify $(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2})$.**

$(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2})$

$= 2\sqrt{3} \cdot \sqrt{3} + 2\sqrt{3} \cdot 2\sqrt{2} - 4\sqrt{2} \cdot \sqrt{3} - 4\sqrt{2} \cdot 2\sqrt{2}$

$= 6 + 4\sqrt{6} - 4\sqrt{6} - 16$

$= -10$

Example 3 **Simplify $\frac{2 - \sqrt{5}}{3 + \sqrt{5}}$.**

$\frac{2 - \sqrt{5}}{3 + \sqrt{5}} = \frac{2 - \sqrt{5}}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}}$

$= \frac{6 - 2\sqrt{5} - 3\sqrt{5} + (\sqrt{5})^2}{3^2 - (\sqrt{5})^2}$

$= \frac{6 - 5\sqrt{5} + 5}{9 - 5}$

$= \frac{11 - 5\sqrt{5}}{4}$

Exercises

Simplify.

1. $3\sqrt{2} + \sqrt{50} - 4\sqrt{8}$

2. $\sqrt{20} + \sqrt{125} - \sqrt{45}$

3. $\sqrt{300} - \sqrt{27} - \sqrt{75}$

4. $\sqrt[3]{81} \cdot \sqrt[3]{24}$

5. $\sqrt[3]{2}(\sqrt[3]{4} + \sqrt[3]{12})$

6. $2\sqrt{3}(\sqrt{15} + \sqrt{60})$

7. $(2 + 3\sqrt{7})(4 + \sqrt{7})$

8. $(6\sqrt{3} - 4\sqrt{2})(3\sqrt{3} + \sqrt{2})$

9. $(4\sqrt{2} - 3\sqrt{5})(2\sqrt{20 + 5})$

10. $\frac{5\sqrt{48} + \sqrt{75}}{5\sqrt{3}}$

11. $\frac{4 + \sqrt{2}}{2 - \sqrt{2}}$

12. $\frac{5 - 3\sqrt{3}}{1 + 2\sqrt{3}}$

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5-6 Skills Practice

Radical Expressions

Lesson 5-6

Simplify.

1. $\sqrt{24}$

2. $\sqrt{75}$

3. $\sqrt[3]{16}$

4. $-\sqrt[4]{48}$

5. $4\sqrt{50x^5}$

6. $\sqrt[4]{64a^4b^4}$

7. $\sqrt[3]{-\frac{1}{8}d^2f^5}$

8. $\sqrt{\frac{25}{36}s^2t}$

9. $-\sqrt{\frac{3}{7}}$

10. $\sqrt[3]{\frac{2}{9}}$

11. $\sqrt{\frac{2g^3}{5z}}$

12. $(3\sqrt{3})(5\sqrt{3})$

13. $(4\sqrt{12})(3\sqrt{20})$

14. $\sqrt{2} + \sqrt{8} + \sqrt{50}$

15. $\sqrt{12} - 2\sqrt{3} + \sqrt{108}$

16. $8\sqrt{5} - \sqrt{45} - \sqrt{80}$

17. $2\sqrt{48} - \sqrt{75} - \sqrt{12}$

18. $(2 + \sqrt{3})(6 - \sqrt{2})$

19. $(1 - \sqrt{5})(1 + \sqrt{5})$

20. $(3 - \sqrt{7})(5 + \sqrt{2})$

21. $(\sqrt{2} - \sqrt{6})^2$

22. $\frac{3}{7 - \sqrt{2}}$

23. $\frac{4}{3 + \sqrt{2}}$

24. $\frac{5}{8 - \sqrt{6}}$

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5-6 Practice

Radical Expressions

Simplify.

1. $\sqrt{540}$

2. $\sqrt[3]{-432}$

3. $\sqrt[3]{128}$

4. $-\sqrt[4]{405}$

5. $\sqrt[3]{-5000}$

6. $\sqrt[5]{-1215}$

7. $\sqrt[3]{125t^6w^2}$

8. $\sqrt[4]{48v^8z^{13}}$

9. $\sqrt[3]{8g^3k^8}$

10. $\sqrt{45x^3y^8}$

11. $\sqrt{\frac{11}{9}}$

12. $\sqrt[3]{\frac{216}{24}}$

13. $\sqrt{\frac{1}{128}c^4d^7}$

14. $\sqrt{\frac{9a^5}{64b^4}}$

15. $\sqrt[4]{\frac{8}{9a^3}}$

16. $(3\sqrt{15})(-4\sqrt{45})$

17. $(2\sqrt{24})(7\sqrt{18})$

18. $\sqrt{810} + \sqrt{240} - \sqrt{250}$

19. $6\sqrt{20} + 8\sqrt{5} - 5\sqrt{45}$

20. $8\sqrt{48} - 6\sqrt{75} + 7\sqrt{80}$

21. $(3\sqrt{2} + 2\sqrt{3})^2$

22. $(3 - \sqrt{7})^2$

23. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

24. $(\sqrt{2} + \sqrt{10})(\sqrt{2} - \sqrt{10})$

25. $(1 + \sqrt{6})(5 - \sqrt{7})$

26. $(\sqrt{3} + 4\sqrt{7})^2$

27. $(\sqrt{108} - 6\sqrt{3})^2$

28. $\frac{\sqrt{3}}{\sqrt{5} - 2}$

29. $\frac{6}{\sqrt{2} - 1}$

30. $\frac{5 + \sqrt{3}}{4 + \sqrt{3}}$

31. $\frac{3 + \sqrt{2}}{2 - \sqrt{2}}$

32. $\frac{3 + \sqrt{6}}{5 - \sqrt{24}}$

33. $\frac{3 + \sqrt{x}}{2 - \sqrt{x}}$

34. BRAKING The formula $s = 2\sqrt{5\ell}$ estimates the speed s in miles per hour of a car when it leaves skid marks ℓ feet long. Use the formula to write a simplified expression for s if $\ell = 85$. Then evaluate s to the nearest mile per hour.

35. PYTHAGOREAN THEOREM The measures of the legs of a right triangle can be represented by the expressions $6x^2y$ and $9x^2y$. Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse.

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5-6 Reading to Learn Mathematics

Radical Expressions

Lesson 5-6

Pre-Activity **How do radical expressions apply to falling objects?**

Read the introduction to Lesson 5-6 at the top of page 250 in your textbook.

Describe how you could use the formula given in your textbook and a calculator to find the time, to the nearest tenth of a second, that it would take for the water balloons to drop 22 feet. (Do not actually calculate the time.)

Reading the Lesson

1. Complete the conditions that must be met for a radical expression to be in simplified form.

- The ________________ n is as ________________ as possible.
- The ________________ contains no ________________ (other than 1) that are nth powers of a(n) ________________ or polynomial.
- The radicand contains no ________________.
- No ________________ appear in the ________________.

2. a. What are conjugates of radical expressions used for?

b. How would you use a conjugate to simplify the radical expression $\frac{1 + \sqrt{2}}{3 - \sqrt{2}}$?

c. In order to simplify the radical expression in part b, two multiplications are necessary. The multiplication in the numerator would be done by the ________________ method, and the multiplication in the denominator would be done by finding the ________________of ________________ ________________.

Helping You Remember

3. One way to remember something is to explain it to another person. When rationalizing the denominator in the expression $\frac{1}{\sqrt[3]{2}}$, many students think they should multiply numerator and denominator by $\frac{\sqrt[3]{2}}{\sqrt[3]{2}}$. How would you explain to a classmate why this is incorrect and what he should do instead.

5-6 Enrichment

Special Products with Radicals

Notice that $(\sqrt{3})(\sqrt{3}) = 3$, or $(\sqrt{3})^2 = 3$.

In general, $(\sqrt{x})^2 = x$ when $x \geq 0$.

Also, notice that $(\sqrt{9})(\sqrt{4}) = \sqrt{36}$.

In general, $(\sqrt{x})(\sqrt{y}) = \sqrt{xy}$ when x and y are not negative.

You can use these ideas to find the special products below.

$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

$(\sqrt{a} + \sqrt{b})^2 = (\sqrt{a})^2 + 2\sqrt{ab} + (\sqrt{b})^2 = a + 2\sqrt{ab} + b$

$(\sqrt{a} - \sqrt{b})^2 = (\sqrt{a})^2 - 2\sqrt{ab} + (\sqrt{b})^2 = a - 2\sqrt{ab} + b$

Example 1 **Find the product: $(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5})$.**

$(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5}) = (\sqrt{2})^2 - (\sqrt{5})^2 = 2 - 5 = -3$

Example 2 **Evaluate $(\sqrt{2} + \sqrt{8})^2$.**

$(\sqrt{2} + \sqrt{8})^2 = (\sqrt{2})^2 + 2\sqrt{2}\sqrt{8} + (\sqrt{8})^2$

$= 2 + 2\sqrt{16} + 8 = 2 + 2(4) + 8 = 2 + 8 + 8 = 18$

Multiply.

1. $(\sqrt{3} - \sqrt{7})(\sqrt{3} + \sqrt{7})$

2. $(\sqrt{10} + \sqrt{2})(\sqrt{10} - \sqrt{2})$

3. $(\sqrt{2x} - \sqrt{6})(\sqrt{2x} - \sqrt{6})$

4. $(\sqrt{3} - 27)^2$

5. $(\sqrt{1000} + \sqrt{10})^2$

6. $(\sqrt{y} + \sqrt{5})(\sqrt{y} - \sqrt{5})$

7. $(\sqrt{50} - \sqrt{x})^2$

8. $(\sqrt{x} + 20)^2$

You can extend these ideas to patterns for sums and differences of cubes.
Study the pattern below.

$(\sqrt[3]{8} - \sqrt[3]{x})(\sqrt[3]{8^2} + \sqrt[3]{8x} + \sqrt[3]{x^2}) = \sqrt[3]{8^3} - \sqrt[3]{x^3} = 8 - x$

Multiply.

9. $(\sqrt[3]{2} - \sqrt[3]{5})(\sqrt[3]{2^2} + \sqrt[3]{10} + \sqrt[3]{5^2})$

10. $(\sqrt[3]{y} + \sqrt[3]{w})(\sqrt[3]{y^2} - \sqrt[3]{yw} + \sqrt[3]{w^2})$

11. $(\sqrt[3]{7} + \sqrt[3]{20})(\sqrt[3]{7^2} - \sqrt[3]{140} + \sqrt[3]{20^2})$

12. $(\sqrt[3]{11} - \sqrt[3]{8})(\sqrt[3]{11^2} + \sqrt[3]{88} + \sqrt[3]{8^2})$

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5-7 Study Guide and Intervention

Rational Exponents

Rational Exponents and Radicals

Definition of $b^{\frac{1}{n}}$	For any real number *b* and any positive integer *n*, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and *n* is even.
Definition of $b^{\frac{m}{n}}$	For any nonzero real number *b*, and any integers *m* and *n*, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$, except when $b < 0$ and *n* is even.

Example 1 **Write $28^{\frac{1}{2}}$ in radical form.**

Notice that $28 > 0$.

$$28^{\frac{1}{2}} = \sqrt{28} = \sqrt{2^2 \cdot 7} = \sqrt{2^2} \cdot \sqrt{7} = 2\sqrt{7}$$

Example 2 **Evaluate $\left(\frac{-8}{-125}\right)^{\frac{1}{3}}$.**

Notice that $-8 < 0$, $-125 < 0$, and 3 is odd.

$$\left(\frac{-8}{-125}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{-8}}{\sqrt[3]{-125}} = \frac{-2}{-5} = \frac{2}{5}$$

Exercises

Write each expression in radical form.

1. $11^{\frac{1}{7}}$

2. $15^{\frac{1}{3}}$

3. $300^{\frac{3}{2}}$

Write each radical using rational exponents.

4. $\sqrt{47}$

5. $\sqrt[3]{3a^5b^2}$

6. $\sqrt[4]{162p^5}$

Evaluate each expression.

7. $-27^{\frac{2}{3}}$

8. $\dfrac{5^{-\frac{1}{2}}}{2\sqrt{5}}$

9. $(0.0004)^{\frac{1}{2}}$

10. $8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}}$

11. $\dfrac{144^{-\frac{1}{2}}}{27^{-\frac{1}{3}}}$

12. $\dfrac{16^{-\frac{1}{2}}}{(0.25)^{\frac{1}{2}}}$

Lesson 5-7

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5-7 Study Guide and Intervention *(continued)*

Rational Exponents

Simplify Expressions All the properties of powers from Lesson 5-1 apply to rational exponents. When you simplify expressions with rational exponents, leave the exponent in rational form, and write the expression with all positive exponents. Any exponents in the denominator must be positive integers

When you simplify radical expressions, you may use rational exponents to simplify, but your answer should be in radical form. Use the smallest index possible.

Example 1 **Simplify $y^{\frac{2}{3}} \cdot y^{\frac{3}{8}}$.**

$y^{\frac{2}{3}} \cdot y^{\frac{3}{8}} = y^{\frac{2}{3} + \frac{3}{8}} = y^{\frac{25}{24}}$

Example 2 **Simplify $\sqrt[4]{144x^6}$.**

$$\sqrt[4]{144x^6} = (144x^6)^{\frac{1}{4}}$$
$$= (2^4 \cdot 3^2 \cdot x^6)^{\frac{1}{4}}$$
$$= (2^4)^{\frac{1}{4}} \cdot (3^2)^{\frac{1}{4}} \cdot (x^6)^{\frac{1}{4}}$$
$$= 2 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{3}{2}} = 2x \cdot (3x)^{\frac{1}{2}} = 2x\sqrt{3x}$$

Exercises

Simplify each expression.

1. $x^{\frac{4}{5}} \cdot x^{\frac{6}{5}}$

2. $\left(y^{\frac{2}{3}}\right)^{\frac{3}{4}}$

3. $p^{\frac{4}{5}} \cdot p^{\frac{7}{10}}$

4. $\left(m^{-\frac{6}{5}}\right)^{\frac{2}{5}}$

5. $x^{-\frac{3}{8}} \cdot x^{\frac{4}{3}}$

6. $\left(s^{-\frac{1}{6}}\right)^{-\frac{4}{3}}$

7. $\dfrac{p}{p^{\frac{1}{3}}}$

8. $\left(a^{\frac{2}{3}}\right)^{\frac{6}{5}} \cdot \left(a^{\frac{2}{5}}\right)^{3}$

9. $\dfrac{x^{-\frac{1}{2}}}{x^{-\frac{1}{3}}}$

10. $\sqrt[6]{128}$

11. $\sqrt[4]{49}$

12. $\sqrt[5]{288}$

13. $\sqrt{32} \cdot 3\sqrt{16}$

14. $\sqrt[3]{25} \cdot \sqrt{125}$

15. $\sqrt[6]{16}$

16. $\dfrac{x - \sqrt[3]{3}}{\sqrt{12}}$

17. $\sqrt{\sqrt[3]{48}}$

18. $\dfrac{a\sqrt[3]{b^4}}{\sqrt{ab^3}}$

5-7 Skills Practice

Rational Exponents

Write each expression in radical form.

1. $3^{\frac{1}{6}}$

2. $8^{\frac{1}{5}}$

3. $12^{\frac{2}{3}}$

4. $(s^3)^{\frac{3}{5}}$

Write each radical using rational exponents.

5. $\sqrt{51}$

6. $\sqrt[3]{37}$

7. $\sqrt[4]{15^3}$

8. $\sqrt[3]{6xy^2}$

Evaluate each expression.

9. $32^{\frac{1}{5}}$

10. $81^{\frac{1}{4}}$

11. $27^{-\frac{1}{3}}$

12. $4^{-\frac{1}{2}}$

13. $16^{\frac{3}{2}}$

14. $(-243)^{\frac{4}{5}}$

15. $27^{\frac{1}{3}} \cdot 27^{\frac{5}{3}}$

16. $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

Simplify each expression.

17. $c^{\frac{12}{5}} \cdot c^{\frac{3}{5}}$

18. $m^{\frac{2}{9}} \cdot m^{\frac{16}{9}}$

19. $\left(q^{\frac{1}{2}}\right)^3$

20. $p^{-\frac{1}{5}}$

21. $x^{-\frac{6}{11}}$

22. $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$

23. $\dfrac{y^{-\frac{1}{2}}}{y^{\frac{1}{4}}}$

24. $\dfrac{n^{\frac{1}{3}}}{n^{\frac{1}{6}} \cdot n^{\frac{1}{2}}}$

25. $\sqrt[12]{64}$

26. $\sqrt[8]{49a^8b^2}$

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5-7 Practice

Rational Exponents

Write each expression in radical form.

1. $5^{\frac{1}{3}}$ **2.** $6^{\frac{2}{5}}$ **3.** $m^{\frac{4}{7}}$ **4.** $(n^3)^{\frac{2}{5}}$

Write each radical using rational exponents.

5. $\sqrt{79}$ **6.** $\sqrt[4]{153}$ **7.** $\sqrt[3]{27m^6n^4}$ **8.** $5\sqrt{2a^{10}b}$

Evaluate each expression.

9. $81^{\frac{1}{4}}$ **10.** $1024^{-\frac{1}{5}}$ **11.** $8^{-\frac{5}{3}}$

12. $-256^{-\frac{3}{4}}$ **13.** $(-64)^{-\frac{2}{3}}$ **14.** $27^{\frac{1}{3}} \cdot 27^{\frac{4}{3}}$

15. $\left(\frac{125}{216}\right)^{\frac{2}{3}}$ **16.** $\dfrac{64^{\frac{2}{3}}}{343^{\frac{2}{3}}}$ **17.** $\left(25^{\frac{1}{2}}\right)\left(-64^{-\frac{1}{3}}\right)$

Simplify each expression.

18. $g^{\frac{4}{7}} \cdot g^{\frac{3}{7}}$ **19.** $s^{\frac{3}{4}} \cdot s^{\frac{13}{4}}$ **20.** $\left(u^{-\frac{1}{3}}\right)^{-\frac{4}{5}}$ **21.** $y^{-\frac{1}{2}}$

22. $b^{-\frac{3}{5}}$ **23.** $\dfrac{q^{\frac{3}{5}}}{q^{\frac{2}{5}}}$ **24.** $\dfrac{t^{\frac{2}{3}}}{5t^{\frac{1}{2}} \cdot t^{-\frac{3}{4}}}$ **25.** $\dfrac{2z^{\frac{1}{2}}}{z^{\frac{1}{2}} - 1}$

26. $\sqrt[10]{8^5}$ **27.** $\sqrt{12} \cdot \sqrt[5]{12^3}$ **28.** $\sqrt[4]{6} \cdot 3\sqrt[4]{6}$ **29.** $\dfrac{a}{\sqrt{3b}}$

30. ELECTRICITY The amount of current in amperes I that an appliance uses can be calculated using the formula $I = \left(\frac{P}{R}\right)^{\frac{1}{2}}$, where P is the power in watts and R is the resistance in ohms. How much current does an appliance use if $P = 500$ watts and $R = 10$ ohms? Round your answer to the nearest tenth.

31. BUSINESS A company that produces DVDs uses the formula $C = 88n^{\frac{1}{3}} + 330$ to calculate the cost C in dollars of producing n DVDs per day. What is the company's cost to produce 150 DVDs per day? Round your answer to the nearest dollar.

5-7 Reading to Learn Mathematics

Rational Exponents

Pre-Activity **How do rational exponents apply to astronomy?**

Read the introduction to Lesson 5-7 at the top of page 257 in your textbook. The formula in the introduction contains the exponent $\frac{2}{5}$. What do you think it might mean to raise a number to the $\frac{2}{5}$ power?

Reading the Lesson

1. Complete the following definitions of rational exponents.

- For any real number b and for any positive integer n, $b^{\frac{1}{n}}$ = ________________ except when b ________________ and n is ________________.
- For any nonzero real number b, and any integers m and n, with n ________________, $b^{\frac{m}{n}}$ = ________________ = ________________, except when b ________________ and n is ________________.

2. Complete the conditions that must be met in order for an expression with rational exponents to be simplified.

- It has no ________________ exponents.
- It has no ________________ exponents in the ________________.
- It is not a ________________ fraction.
- The ________________ of any remaining ________________ is the ________________ number possible.

3. Margarita and Pierre were working together on their algebra homework. One exercise asked them to evaluate the expression $27^{\frac{4}{3}}$. Margarita thought that they should raise 27 to the fourth power first and then take the cube root of the result. Pierre thought that they should take the cube root of 27 first and then raise the result to the fourth power. Whose method is correct?

Helping You Remember

4. Some students have trouble remembering which part of the fraction in a rational exponent gives the power and which part gives the root. How can your knowledge of integer exponents help you to keep this straight?

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5-7 Enrichment

Lesser-Known Geometric Formulas

Many geometric formulas involve radical expressions.

Make a drawing to illustrate each of the formulas given on this page. Then evaluate the formula for the given value of the variable. Round answers to the nearest hundredth.

1. The area of an isosceles triangle. Two sides have length a; the other side has length c. Find A when $a = 6$ and $c = 7$.

$A = \frac{c}{4}\sqrt{4a^2 - c^2}$

2. The area of an equilateral triangle with a side of length a. Find A when $a = 8$.

$A = \frac{a^2}{4}\sqrt{3}$

3. The area of a regular pentagon with a side of length a. Find A when $a = 4$.

$A = \frac{a^2}{4}\sqrt{25 + 10\sqrt{5}}$

4. The area of a regular hexagon with a side of length a. Find A when $a = 9$.

$A = \frac{3a^2}{2}\sqrt{3}$

5. The volume of a regular tetrahedron with an edge of length a. Find V when $a = 2$.

$V = \frac{a^3}{12}\sqrt{2}$

6. The area of the curved surface of a right cone with an altitude of h and radius of base r. Find S when $r = 3$ and $h = 6$.

$S = \pi r\sqrt{r^2 + h^2}$

7. Heron's Formula for the area of a triangle uses the semi-perimeter s, where $s = \frac{a + b + c}{2}$. The sides of the triangle have lengths a, b, and c. Find A when $a = 3$, $b = 4$, and $c = 5$.

$A = \sqrt{s(s - a)(s - b)(s - c)}$

8. The radius of a circle inscribed in a given triangle also uses the semi-perimeter. Find r when $a = 6$, $b = 7$, and $c = 9$.

$r = \frac{\sqrt{s(s - a)(s - b)(s - c)}}{s}$

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5-8 Study Guide and Intervention

Radical Equations and Inequalities

Solve Radical Equations The following steps are used in solving equations that have variables in the radicand. Some algebraic procedures may be needed before you use these steps.

Step 1	Isolate the radical on one side of the equation.
Step 2	To eliminate the radical, raise each side of the equation to a power equal to the index of the radical.
Step 3	Solve the resulting equation.
Step 4	Check your solution in the original equation to make sure that you have not obtained any extraneous roots.

Example 1 **Solve $2\sqrt{4x + 8} - 4 = 8$.**

$2\sqrt{4x + 8} - 4 = 8$	Original equation
$2\sqrt{4x + 8} = 12$	Add 4 to each side.
$\sqrt{4x + 8} = 6$	Isolate the radical.
$4x + 8 = 36$	Square each side.
$4x = 28$	Subtract 8 from each side.
$x = 7$	Divide each side by 4.

Check

$2\sqrt{4(7) + 8} - 4 \stackrel{?}{=} 8$

$2\sqrt{36} - 4 \stackrel{?}{=} 8$

$2(6) - 4 \stackrel{?}{=} 8$

$8 = 8$

The solution $x = 7$ checks.

Example 2 **Solve $\sqrt{3x + 1} = \sqrt{5x} - 1$.**

$\sqrt{3x + 1} = \sqrt{5x} - 1$	Original equation
$3x + 1 = 5x - 2\sqrt{5x} + 1$	Square each side.
$2\sqrt{5x} = 2x$	Simplify.
$\sqrt{5x} = x$	Isolate the radical.
$5x = x^2$	Square each side.
$x^2 - 5x = 0$	Subtract $5x$ from each side.
$x(x - 5) = 0$	Factor.
$x = 0$ or $x = 5$	

Check

$\sqrt{3(0) + 1} = 1$, but $\sqrt{5(0)} - 1 = -1$, so 0 is not a solution.

$\sqrt{3(5) + 1} = 4$, and $\sqrt{5(5)} - 1 = 4$, so the solution is $x = 5$.

Exercises

Solve each equation.

1. $3 + 2x\sqrt{3} = 5$

2. $2\sqrt{3x + 4} + 1 = 15$

3. $8 + \sqrt{x + 1} = 2$

4. $\sqrt{5 - x} - 4 = 6$

5. $12 + \sqrt{2x - 1} = 4$

6. $\sqrt{12 - x} = 0$

7. $\sqrt{21} - \sqrt{5x - 4} = 0$

8. $10 - \sqrt{2x} = 5$

9. $\sqrt{x^2 + 7x} = \sqrt{7x - 9}$

10. $4\sqrt[3]{2x + 11} - 2 = 10$

11. $2\sqrt{x + 11} = \sqrt{x + 2} + \sqrt{3x - 6}$

12. $\sqrt{9x - 11} = x + 1$

Lesson 5-8

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5-8 Study Guide and Intervention *(continued)*

Radical Equations and Inequalities

Solve Radical Inequalities A **radical inequality** is an inequality that has a variable in a radicand. Use the following steps to solve radical inequalities.

Step 1	If the index of the root is even, identify the values of the variable for which the radicand is nonnegative.
Step 2	Solve the inequality algebraically.
Step 3	Test values to check your solution.

Example **Solve $5 - \sqrt{20x + 4} \geq -3$.**

Since the radicand of a square root must be greater than or equal to zero, first solve
$20x + 4 \geq 0$.

$$20x + 4 \geq 0$$
$$20x \geq -4$$
$$x \geq -\frac{1}{5}$$

Now solve $5 - \sqrt{20x + 4} \geq -3$.

$5 - \sqrt{20x + 4} \geq -3$ Original inequality

$\sqrt{20x + 4} \leq 8$ Isolate the radical.

$20x + 4 \leq 64$ Eliminate the radical by squaring each side.

$20x \leq 60$ Subtract 4 from each side.

$x \leq 3$ Divide each side by 20.

It appears that $-\frac{1}{5} \leq x \leq 3$ is the solution. Test some values.

$x = -1$	$x = 0$	$x = 4$
$\sqrt{20(-1) + 4}$ is not a real number, so the inequality is not satisfied.	$5 - \sqrt{20(0) + 4} = 3$, so the inequality is satisfied.	$5 - \sqrt{20(4) + 4} \approx -4.2$, so the inequality is not satisfied

Therefore the solution $-\frac{1}{5} \leq x \leq 3$ checks.

Exercises

Solve each inequality.

1. $\sqrt{c - 2} + 4 \geq 7$

2. $3\sqrt{2x - 1} + 6 < 15$

3. $\sqrt{10x + 9} - 2 > 5$

4. $5\sqrt[3]{x + 2} - 8 < 2$

5. $8 - \sqrt{3x + 4} \geq 3$

6. $\sqrt{2x + 8} - 4 > 2$

7. $9 - \sqrt{6x + 3} \geq 6$

8. $\frac{20}{\sqrt{3x + 1}} \leq 4$

9. $2\sqrt{5x - 6} - 1 < 5$

10. $\sqrt{2x + 12} + 4 \geq 12$

11. $\sqrt{2d + 1} + \sqrt{d} \leq 5$

12. $4\sqrt{b + 3} - \sqrt{b - 2} \geq 10$

5-8 Skills Practice

Radical Equations and Inequalities

Solve each equation or inequality.

1. $\sqrt{x} = 5$

2. $\sqrt{x} + 3 = 7$

3. $5\sqrt{j} = 1$

4. $v^{\frac{1}{2}} + 1 = 0$

5. $18 - 3y^{\frac{1}{2}} = 25$

6. $\sqrt[3]{2w} = 4$

7. $\sqrt{b - 5} = 4$

8. $\sqrt{3n + 1} = 5$

9. $\sqrt[3]{3r - 6} = 3$

10. $2 + \sqrt{3p + 7} = 6$

11. $\sqrt{k - 4} - 1 = 5$

12. $(2d + 3)^{\frac{1}{3}} = 2$

13. $(t - 3)^{\frac{1}{3}} = 2$

14. $4 - (1 - 7u)^{\frac{1}{3}} = 0$

15. $\sqrt{3z - 2} = \sqrt{z - 4}$

16. $\sqrt{g + 1} = \sqrt{2g - 7}$

17. $\sqrt{x - 1} = 4\sqrt{x + 1}$

18. $5 + \sqrt{s - 3} \leq 6$

19. $-2 + \sqrt{3x + 3} < 7$

20. $-\sqrt{2a + 4} \geq -6$

21. $2\sqrt{4r - 3} > 10$

22. $4 - \sqrt{3x + 1} > 3$

23. $\sqrt{y + 4} - 3 \geq 3$

24. $-3\sqrt{11r + 3} \geq -15$

Lesson 5-8

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5-8 Practice

Radical Equations and Inequalities

Solve each equation or inequality.

1. $\sqrt{x} = 8$

2. $4 - \sqrt{x} = 3$

3. $\sqrt{2p} + 3 = 10$

4. $4\sqrt{3h} - 2 = 0$

5. $c^{\frac{1}{2}} + 6 = 9$

6. $18 + 7h^{\frac{1}{2}} = 12$

7. $\sqrt[3]{d + 2} = 7$

8. $\sqrt[5]{w - 7} = 1$

9. $6 + \sqrt[3]{q - 4} = 9$

10. $\sqrt[4]{y - 9} + 4 = 0$

11. $\sqrt{2m - 6} - 16 = 0$

12. $\sqrt[3]{4m + 1} - 2 = 2$

13. $\sqrt{8n - 5} - 1 = 2$

14. $\sqrt{1 - 4t} - 8 = -6$

15. $\sqrt{2t - 5} - 3 = 3$

16. $(7v - 2)^{\frac{1}{4}} + 12 = 7$

17. $(3g + 1)^{\frac{1}{2}} - 6 = 4$

18. $(6u - 5)^{\frac{1}{3}} + 2 = -3$

19. $\sqrt{2d - 5} = \sqrt{d - 1}$

20. $\sqrt{4r - 6} = \sqrt{r}$

21. $\sqrt{6x - 4} = \sqrt{2x + 10}$

22. $\sqrt{2x + 5} = \sqrt{2x + 1}$

23. $3\sqrt{a} \geq 12$

24. $\sqrt{z + 5} + 4 \leq 13$

25. $8 + \sqrt{2q} \leq 5$

26. $\sqrt{2a - 3} < 5$

27. $9 - \sqrt{c + 4} \leq 6$

28. $\sqrt[3]{x - 1} < -2$

29. STATISTICS Statisticians use the formula $\sigma = \sqrt{v}$ to calculate a standard deviation σ, where v is the variance of a data set. Find the variance when the standard deviation is 15.

30. GRAVITATION Helena drops a ball from 25 feet above a lake. The formula $t = \frac{1}{4}\sqrt{25 - h}$ describes the time t in seconds that the ball is h feet above the water. How many feet above the water will the ball be after 1 second?

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5-8 Reading to Learn Mathematics

Radical Equations and Inequalities

Pre-Activity **How do radical equations apply to manufacturing?**

Read the introduction to Lesson 5-8 at the top of page 263 in your textbook.

Explain how you would use the formula in your textbook to find the cost of producing 125,000 computer chips. (Describe the steps of the calculation in the order in which you would perform them, but do not actually do the calculation.)

Reading the Lesson

1. a. What is an *extraneous solution* of a radical equation?

b. Describe two ways you can check the proposed solutions of a radical equation in order to determine whether any of them are extraneous solutions.

2. Complete the steps that should be followed in order to solve a radical inequality.

Step 1 If the _________________ of the root is _________________, identify the values of the variable for which the _________________ is _________________.

Step 2 Solve the _________________ algebraically.

Step 3 Test _________________ to check your solution.

Helping You Remember

3. One way to remember something is to explain it to another person. Suppose that your friend Leora thinks that she does not need to check her solutions to radical equations by substitution because she knows she is very careful and seldom makes mistakes in her work. How can you explain to her that she should nevertheless check every proposed solution in the original equation?

Lesson 5-8

5-8 Enrichment

Truth Tables

In mathematics, the basic operations are addition, subtraction, multiplication, division, finding a root, and raising to a power. In logic, the basic operations are the following: *not* ($\sim$), *and* ($\wedge$), *or* ($\vee$), and *implies* ($\rightarrow$).

If P and Q are statements, then $\sim P$ means not P; $\sim Q$ means not Q; $P \wedge Q$ means P and Q; $P \vee Q$ means P or Q; and $P \rightarrow Q$ means P implies Q. The operations are defined by truth tables. On the left below is the truth table for the statement $\sim P$. Notice that there are two possible conditions for P, true (T) or false (F). If P is true, $\sim P$ is false; if P is false, $\sim P$ is true. Also shown are the truth tables for $P \wedge Q$, $P \vee Q$, and $P \rightarrow Q$.

P	$\sim P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

You can use this information to find out under what conditions a complex statement is true.

Example **Under what conditions is $\sim P \vee Q$ true?**

Create the truth table for the statement. Use the information from the truth table above for $P \vee Q$ to complete the last column.

P	Q	$\sim P$	$\sim P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

The truth table indicates that $\sim P \vee Q$ is true in all cases except where P is true and Q is false.

Use truth tables to determine the conditions under which each statement is true.

1. $\sim P \vee \sim Q$

2. $\sim P \rightarrow (P \rightarrow Q)$

3. $(P \vee Q) \vee (\sim P \wedge \sim Q)$

4. $(P \rightarrow Q) \vee (Q \rightarrow P)$

5. $(P \rightarrow Q) \wedge (Q \rightarrow P)$

6. $(\sim P \wedge \sim Q) \rightarrow \sim(P \vee Q)$

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5-9 Study Guide and Intervention

Complex Numbers

Add and Subtract Complex Numbers

Complex Number	A complex number is any number that can be written in the form $a + b\boldsymbol{i}$, where a and b are real numbers and $\boldsymbol{i}$ is the imaginary unit ($\boldsymbol{i}^2 = -1$). a is called the real part, and b is called the imaginary part.
Addition and Subtraction of Complex Numbers	Combine like terms. $(a + b\boldsymbol{i}) + (c + d\boldsymbol{i}) = (a + c) + (b + d)\boldsymbol{i}$ $(a + b\boldsymbol{i}) - (c + d\boldsymbol{i}) = (a - c) + (b - d)\boldsymbol{i}$

Example 1 **Simplify $(6 + i) + (4 - 5i)$.**

$(6 + \boldsymbol{i}) + (4 - 5\boldsymbol{i})$
$= (6 + 4) + (1 - 5)\boldsymbol{i}$
$= 10 - 4\boldsymbol{i}$

Example 2 **Simplify $(8 + 3i) - (6 - 2i)$.**

$(8 + 3\boldsymbol{i}) - (6 - 2\boldsymbol{i})$
$= (8 - 6) + [3 - (-2)]\boldsymbol{i}$
$= 2 + 5\boldsymbol{i}$

To solve a quadratic equation that does not have real solutions, you can use the fact that $\boldsymbol{i}^2 = -1$ to find complex solutions.

Example 3 **Solve $2x^2 + 24 = 0$.**

$2x^2 + 24 = 0$	Original equation
$2x^2 = -24$	Subtract 24 from each side.
$x^2 = -12$	Divide each side by 2.
$x = \pm\sqrt{-12}$	Take the square root of each side.
$x = \pm 2\boldsymbol{i}\sqrt{3}$	$\sqrt{-12} = \sqrt{4} \cdot \sqrt{-1} \cdot \sqrt{3}$

Exercises

Simplify.

1. $(-4 + 2\boldsymbol{i}) + (6 - 3\boldsymbol{i})$

2. $(5 - \boldsymbol{i}) - (3 - 2\boldsymbol{i})$

3. $(6 - 3\boldsymbol{i}) + (4 - 2\boldsymbol{i})$

4. $(-11 + 4\boldsymbol{i}) - (1 - 5\boldsymbol{i})$

5. $(8 + 4\boldsymbol{i}) + (8 - 4\boldsymbol{i})$

6. $(5 + 2\boldsymbol{i}) - (-6 - 3\boldsymbol{i})$

7. $(12 - 5\boldsymbol{i}) - (4 + 3\boldsymbol{i})$

8. $(9 + 2\boldsymbol{i}) + (-2 + 5\boldsymbol{i})$

9. $(15 - 12\boldsymbol{i}) + (11 - 13\boldsymbol{i})$

10. $\boldsymbol{i}^4$

11. $\boldsymbol{i}^6$

12. $\boldsymbol{i}^{15}$

Solve each equation.

13. $5x^2 + 45 = 0$

14. $4x^2 + 24 = 0$

15. $-9x^2 = 9$

Lesson 5-9

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5-9 Study Guide and Intervention *(continued)*

Complex Numbers

Multiply and Divide Complex Numbers

Multiplication of Complex Numbers	Use the definition of i^2 and the FOIL method: $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

To divide by a complex number, first multiply the dividend and divisor by the **complex conjugate** of the divisor.

Complex Conjugate	$a + bi$ and $a - bi$ are complex conjugates. The product of complex conjugates is always a real number.

Example 1 **Simplify $(2 - 5i) \cdot (-4 + 2i)$.**

$(2 - 5i) \cdot (-4 + 2i)$

$= 2(-4) + 2(2i) + (-5i)(-4) + (-5i)(2i)$ FOIL

$= -8 + 4i + 20i - 10i^2$ Multiply.

$= -8 + 24i - 10(-1)$ Simplify.

$= 2 + 24i$ Standard form

Example 2 **Simplify $\frac{3 - i}{2 + 3i}$.**

$\frac{3 - i}{2 + 3i} = \frac{3 - i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i}$ Use the complex conjugate of the divisor.

$= \frac{6 - 9i - 2i + 3i^2}{4 - 9i^2}$ Multiply.

$= \frac{3 - 11i}{13}$ $i^2 = -1$

$= \frac{3}{13} - \frac{11}{13}i$ Standard form

Exercises

Simplify.

1. $(2 + i)(3 - i)$ **2.** $(5 - 2i)(4 - i)$ **3.** $(4 - 2i)(1 - 2i)$

4. $(4 - 6i)(2 + 3i)$ **5.** $(2 + i)(5 - i)$ **6.** $(5 - 3i)(-1 - i)$

7. $(1 - i)(2 + 2i)(3 - 3i)$ **8.** $(4 - i)(3 - 2i)(2 + i)$ **9.** $(5 - 2i)(1 - i)(3 + i)$

10. $\frac{5}{3 + i}$ **11.** $\frac{7 - 13i}{2i}$ **12.** $\frac{6 - 5i}{3i}$

13. $\frac{4 - 2i}{3 + i}$ **14.** $\frac{-5 - 3i}{2 - 2i}$ **15.** $\frac{3 + 4i}{4 - 5i}$

16. $\frac{3 + i\sqrt{5}}{3 - i\sqrt{5}}$ **17.** $\frac{4 - i\sqrt{2}}{i\sqrt{2}}$ **18.** $\frac{\sqrt{6} + i\sqrt{3}}{\sqrt{2} - i}$

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5-9 Skills Practice

Complex Numbers

Simplify.

1. $\sqrt{-36}$

2. $\sqrt{-196}$

3. $\sqrt{-81x^6}$

4. $\sqrt{-23} \cdot \sqrt{-46}$

5. $(3\boldsymbol{i})(-2\boldsymbol{i})(5\boldsymbol{i})$

6. $\boldsymbol{i}^{11}$

7. $\boldsymbol{i}^{65}$

8. $(7 - 8\boldsymbol{i}) + (-12 - 4\boldsymbol{i})$

9. $(-3 + 5\boldsymbol{i}) + (18 - 7\boldsymbol{i})$

10. $(10 - 4\boldsymbol{i}) - (7 + 3\boldsymbol{i})$

11. $(2 + \boldsymbol{i})(2 + 3\boldsymbol{i})$

12. $(2 + \boldsymbol{i})(3 - 5\boldsymbol{i})$

13. $(7 - 6\boldsymbol{i})(2 - 3\boldsymbol{i})$

14. $(3 + 4\boldsymbol{i})(3 - 4\boldsymbol{i})$

15. $\frac{8 - 6\boldsymbol{i}}{3\boldsymbol{i}}$

16. $\frac{3\boldsymbol{i}}{4 + 2\boldsymbol{i}}$

Solve each equation.

17. $3x^2 + 3 = 0$

18. $5x^2 + 125 = 0$

19. $4x^2 + 20 = 0$

20. $-x^2 - 16 = 0$

21. $x^2 + 18 = 0$

22. $8x^2 + 96 = 0$

Find the values of *m* and *n* that make each equation true.

23. $20 - 12\boldsymbol{i} = 5m + 4n\boldsymbol{i}$

24. $m - 16\boldsymbol{i} = 3 - 2n\boldsymbol{i}$

25. $(4 + m) + 2n\boldsymbol{i} = 9 + 14\boldsymbol{i}$

26. $(3 - n) + (7m - 14)\boldsymbol{i} = 1 + 7\boldsymbol{i}$

Lesson 5-9

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5-9 Practice

Complex Numbers

Simplify.

1. $\sqrt{-49}$

2. $6\sqrt{-12}$

3. $\sqrt{-121s^8}$

4. $\sqrt{-36a^3b^4}$

5. $\sqrt{-8} \cdot \sqrt{-32}$

6. $\sqrt{-15} \cdot \sqrt{-25}$

7. $(-3\boldsymbol{i})(4\boldsymbol{i})(-5\boldsymbol{i})$

8. $(7\boldsymbol{i})^2(6\boldsymbol{i})$

9. $\boldsymbol{i}^{42}$

10. $\boldsymbol{i}^{55}$

11. $\boldsymbol{i}^{89}$

12. $(5 - 2\boldsymbol{i}) + (-13 - 8\boldsymbol{i})$

13. $(7 - 6\boldsymbol{i}) + (9 + 11\boldsymbol{i})$

14. $(-12 + 48\boldsymbol{i}) + (15 + 21\boldsymbol{i})$

15. $(10 + 15\boldsymbol{i}) - (48 - 30\boldsymbol{i})$

16. $(28 - 4\boldsymbol{i}) - (10 - 30\boldsymbol{i})$

17. $(6 - 4\boldsymbol{i})(6 + 4\boldsymbol{i})$

18. $(8 - 11\boldsymbol{i})(8 - 11\boldsymbol{i})$

19. $(4 + 3\boldsymbol{i})(2 - 5\boldsymbol{i})$

20. $(7 + 2\boldsymbol{i})(9 - 6\boldsymbol{i})$

21. $\frac{6 + 5\boldsymbol{i}}{-2\boldsymbol{i}}$

22. $\frac{2}{7 - 8\boldsymbol{i}}$

23. $\frac{3 - \boldsymbol{i}}{2 - \boldsymbol{i}}$

24. $\frac{2 - 4\boldsymbol{i}}{1 + 3\boldsymbol{i}}$

Solve each equation.

25. $5n^2 + 35 = 0$

26. $2m^2 + 10 = 0$

27. $4m^2 + 76 = 0$

28. $-2m^2 - 6 = 0$

29. $-5m^2 - 65 = 0$

30. $\frac{3}{4}x^2 + 12 = 0$

Find the values of *m* and *n* that make each equation true.

31. $15 - 28\boldsymbol{i} = 3m + 4n\boldsymbol{i}$

32. $(6 - m) + 3n\boldsymbol{i} = -12 + 27\boldsymbol{i}$

33. $(3m + 4) + (3 - n)\boldsymbol{i} = 16 - 3\boldsymbol{i}$

34. $(7 + n) + (4m - 10)\boldsymbol{i} = 3 - 6\boldsymbol{i}$

35. ELECTRICITY The impedance in one part of a series circuit is $1 + 3\boldsymbol{j}$ ohms and the impedance in another part of the circuit is $7 - 5\boldsymbol{j}$ ohms. Add these complex numbers to find the total impedance in the circuit.

36. ELECTRICITY Using the formula $E = IZ$, find the voltage E in a circuit when the current I is $3 - \boldsymbol{j}$ amps and the impedance Z is $3 + 2\boldsymbol{j}$ ohms.

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5-9

Reading to Learn Mathematics

Complex Numbers

Pre-Activity **How do complex numbers apply to polynomial equations?**

Read the introduction to Lesson 5-9 at the top of page 270 in your textbook.

Suppose the number i is defined such that $\boldsymbol{i}^2 = -1$. Complete each equation.

$2\boldsymbol{i}^2 =$ ________ $(2\boldsymbol{i})^2 =$ ________ $\boldsymbol{i}^4 =$ ________

Reading the Lesson

1. Complete each statement.

a. The form $a + b\boldsymbol{i}$ is called the ____________________ of a complex number.

b. In the complex number $4 + 5\boldsymbol{i}$, the real part is ______ and the imaginary part is ______.

This is an example of a complex number that is also a(n) ________________ number.

c. In the complex number 3, the real part is ______ and the imaginary part is ______.

This is example of complex number that is also a(n) ________________ number.

d. In the complex number $7\boldsymbol{i}$, the real part is ______ and the imaginary part is ______.

This is an example of a complex number that is also a(n) ________________ number.

2. Give the complex conjugate of each number.

a. $3 + 7\boldsymbol{i}$ ____________

b. $2 - \boldsymbol{i}$ ____________

3. Why are complex conjugates used in dividing complex numbers?

4. Explain how you would use complex conjugates to find $(3 + 7\boldsymbol{i}) \div (2 - \boldsymbol{i})$.

Helping You Remember

5. How can you use what you know about simplifying an expression such as $\frac{1 + \sqrt{3}}{2 - \sqrt{5}}$ to help you remember how to simplify fractions with imaginary numbers in the denominator?

Lesson 5-9

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5-9 Enrichment

Conjugates and Absolute Value

When studying complex numbers, it is often convenient to represent a complex number by a single variable. For example, we might let $z = x + y\boldsymbol{i}$. We denote the conjugate of z by $\bar{z}$. Thus, $\bar{z} = x - y\boldsymbol{i}$.

We can define the absolute value of a complex number as follows.

$$|z| = |x + y\boldsymbol{i}| = \sqrt{x^2 + y^2}$$

There are many important relationships involving conjugates and absolute values of complex numbers.

Example 1 **Show $|z|^2 = z\bar{z}$ for any complex number z.**

Let $z = x + y\boldsymbol{i}$. Then,

$$\begin{aligned} z &= (x + y\boldsymbol{i})(x - y\boldsymbol{i}) \\ &= x^2 + y^2 \\ &= \sqrt{(x^2 + y^2)^2} \\ &= |z|^2 \end{aligned}$$

Example 2 **Show $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse for any nonzero complex number z.**

We know $|z|^2 = z\bar{z}$. If $z \neq 0$, then we have $z\left(\frac{\bar{z}}{|z|^2}\right) = 1$.

Thus, $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse of z.

For each of the following complex numbers, find the absolute value and multiplicative inverse.

1. $2\boldsymbol{i}$

2. $-4 - 3\boldsymbol{i}$

3. $12 - 5\boldsymbol{i}$

4. $5 - 12\boldsymbol{i}$

5. $1 + \boldsymbol{i}$

6. $\sqrt{3} - \boldsymbol{i}$

7. $\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3}\boldsymbol{i}$

8. $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\boldsymbol{i}$

9. $\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$