GLENCOE MATHEMATICS

Algebra 2

Chapter 7 Resource Masters

Glencoe McGraw-Hill

New York, New York
Columbus, Ohio
Chicago, Illinois
Peoria, Illinois
Woodland Hills, California

Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-828029-X
Skills Practice Workbook	0-07-828023-0
Practice Workbook	0-07-828024-9

ANSWERS FOR WORKBOOKS The answers for Chapter 7 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Send all inquiries to:
The McGraw-Hill Companies
8787 Orion Place
Columbus, OH 43240-4027

ISBN: 0-07-828010-9

Algebra 2
Chapter 7 Resource Masters

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Contents

Teacher's Guide to Using the Chapter 7 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 7 Resource Masters* includes the core materials needed for Chapter 7. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 2 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 7-1. Encourage them to add these pages to their Algebra 2 Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in *Algebra 2* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 7 Resource Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 2. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 406–407. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

7 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 7. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
composition of functions		
depressed polynomial		
end behavior		
Factor Theorem		
Fundamental Theorem of Algebra		
inverse function		
inverse relation		
leading coefficients		
location principle		
one-to-one		

(continued on the next page)

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7

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
polynomial function		
polynomial in one variable		
power function		
quadratic form		
Rational Zero Theorem		
relative maximum		
relative minimum		
remainder theorem		
square root function		
synthetic substitution sihn·THEH·tihk		

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7-1 Study Guide and Intervention

Polynomial Functions

Polynomial Functions

Polynomial in One Variable	A polynomial of degree n in one variable x is an expression of the form $a_0x^n + a_1x^{n-1} + \ldots + a_{n-2}x^2 + a_{n-1}x + a_n$, where the coefficients $a_0, a_1, a_2, \ldots, a_n$ represent real numbers, a_0 is not zero, and n represents a nonnegative integer.

The **degree of a polynomial** in one variable is the greatest exponent of its variable. The **leading coefficient** is the coefficient of the term with the highest degree.

Polynomial Function	A polynomial function of degree n can be described by an equation of the form $P(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-2}x^2 + a_{n-1}x + a_n$, where the coefficients $a_0, a_1, a_2, \ldots, a_n$ represent real numbers, a_0 is not zero, and n represents a nonnegative integer.

Lesson 7-1

Example 1 **What are the degree and leading coefficient of $3x^2 - 2x^4 - 7 + x^3$?**

Rewrite the expression so the powers of x are in decreasing order.

$-2x^4 + x^3 + 3x^2 - 7$

This is a polynomial in one variable. The degree is 4, and the leading coefficient is −2.

Example 2 **Find $f(-5)$ if $f(x) = x^3 + 2x^2 - 10x + 20$.**

$f(x) = x^3 + 2x^2 - 10x + 20$ Original function

$f(-5) = (-5)^3 + 2(-5)^2 - 10(-5) + 20$ Replace x with −5.

$= -125 + 50 + 50 + 20$ Evaluate.

$= -5$ Simplify.

Example 3 **Find $g(a^2 - 1)$ if $g(x) = x^2 + 3x - 4$.**

$g(x) = x^2 + 3x - 4$ Original function

$g(a^2 - 1) = (a^2 - 1)^2 + 3(a^2 - 1) - 4$ Replace x with $a^2 - 1$.

$= a^4 - 2a^2 + 1 + 3a^2 - 3 - 4$ Evaluate.

$= a^4 + a^2 - 6$ Simplify.

Exercises

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $3x^4 + 6x^3 - x^2 + 12$

2. $100 - 5x^3 + 10x^7$

3. $4x^6 + 6x^4 + 8x^8 - 10x^2 + 20$

4. $4x^2 - 3xy + 16y^2$

5. $8x^3 - 9x^5 + 4x^2 - 36$

6. $\frac{x^2}{18} - \frac{x^6}{25} + \frac{x^3}{36} - \frac{1}{72}$

Find $f(2)$ and $f(-5)$ for each function.

7. $f(x) = x^2 - 9$

8. $f(x) = 4x^3 - 3x^2 + 2x - 1$

9. $f(x) = 9x^3 - 4x^2 + 5x + 7$

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7-1 Study Guide and Intervention *(continued)*

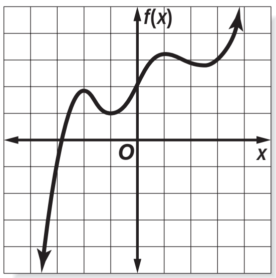
Polynomial Functions

Graphs of Polynomial Functions

End Behavior of Polynomial Functions	If the degree is even and the leading coefficient is positive, then
	$f(x) \to +\infty$ as $x \to -\infty$
	$f(x) \to +\infty$ as $x \to +\infty$
	If the degree is even and the leading coefficient is negative, then
	$f(x) \to -\infty$ as $x \to -\infty$
	$f(x) \to -\infty$ as $x \to +\infty$
	If the degree is odd and the leading coefficient is positive, then
	$f(x) \to -\infty$ as $x \to -\infty$
	$f(x) \to +\infty$ as $x \to +\infty$
	If the degree is odd and the leading coefficient is negative, then
	$f(x) \to +\infty$ as $x \to -\infty$
	$f(x) \to -\infty$ as $x \to +\infty$

Real Zeros of a Polynomial Function	The maximum number of zeros of a polynomial function is equal to the degree of the polynomial. A zero of a function is a point at which the graph intersects the *x*-axis. On a graph, count the number of real zeros of the function by counting the number of times the graph crosses or touches the *x*-axis.

Example **Determine whether the graph represents an odd-degree polynomial or an even-degree polynomial. Then state the number of real zeros.**

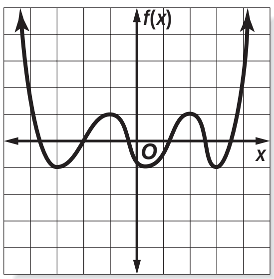


As $x \to -\infty$, $f(x) \to -\infty$ and as $x \to +\infty$, $f(x) \to +\infty$, so it is an odd-degree polynomial function. The graph intersects the *x*-axis at 1 point, so the function has 1 real zero.

Exercises

Determine whether each graph represents an odd-degree polynomial or an even-degree polynomial. Then state the number of real zeros.

1.



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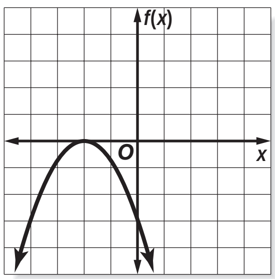

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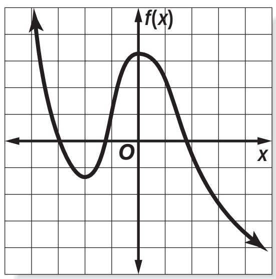


NAME ____________________ DATE __________ PERIOD _____

7-1 Skills Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $a + 8$

2. $(2x - 1)(4x^2 + 3)$

3. $-5x^5 + 3x^3 - 8$

4. $18 - 3y + 5y^2 - y^5 + 7y^6$

5. $u^3 + 4u^2v^2 + v^4$

6. $2r - r^2 + \frac{1}{r^2}$

Find $p(-1)$ and $p(2)$ for each function.

7. $p(x) = 4 - 3x$

8. $p(x) = 3x + x^2$

9. $p(x) = 2x^2 - 4x + 1$

10. $p(x) = -2x^3 + 5x + 3$

11. $p(x) = x^4 + 8x^2 - 10$

12. $p(x) = \frac{1}{3}x^2 - \frac{2}{3}x + 2$

If $p(x) = 4x^2 - 3$ and $r(x) = 1 + 3x$, find each value.

13. $p(a)$

14. $r(2a)$

15. $3r(a)$

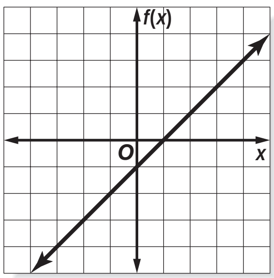
16. $-4p(a)$

17. $p(a^2)$

18. $r(x + 2)$

For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree polynomial function, and
c. state the number of real zeroes.

19.



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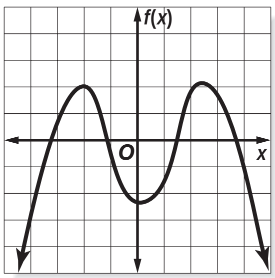

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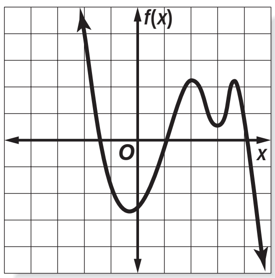

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7-1 Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $(3x^2 + 1)(2x^2 - 9)$

2. $\frac{1}{5}a^3 - \frac{3}{5}a^2 + \frac{4}{5}a$

3. $\frac{2}{m^2} + 3m - 12$

4. $27 + 3xy^3 - 12x^2y^2 - 10y$

Find $p(-2)$ and $p(3)$ for each function.

5. $p(x) = x^3 - x^5$

6. $p(x) = -7x^2 + 5x + 9$

7. $p(x) = -x^5 + 4x^3$

8. $p(x) = 3x^3 - x^2 + 2x - 5$

9. $p(x) = x^4 + \frac{1}{2}x^3 - \frac{1}{2}x$

10. $p(x) = \frac{1}{3}x^3 + \frac{2}{3}x^2 + 3x$

If $p(x) = 3x^2 - 4$ and $r(x) = 2x^2 - 5x + 1$, find each value.

11. $p(8a)$

12. $r(a^2)$

13. $-5r(2a)$

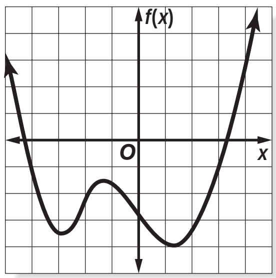
14. $r(x + 2)$

15. $p(x^2 - 1)$

16. $5[p(x + 2)]$

For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree polynomial function, and
c. state the number of real zeroes.

17.



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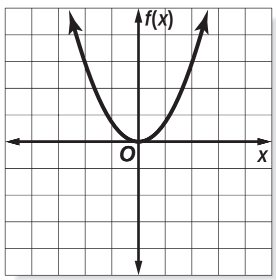

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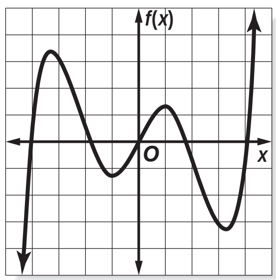

20. WIND CHILL The function $C(s) = 0.013s^2 - s - 7$ estimates the wind chill temperature $C(s)$ at 0°F for wind speeds s from 5 to 30 miles per hour. Estimate the wind chill temperature at 0°F if the wind speed is 20 miles per hour.

7-1 Reading to Learn Mathematics

Polynomial Functions

Pre-Activity **Where are polynomial functions found in nature?**

Read the introduction to Lesson 7-1 at the top of page 346 in your textbook.

- In the honeycomb cross section shown in your textbook, there is 1 hexagon in the center, 6 hexagons in the second ring, and 12 hexagons in the third ring. How many hexagons will there be in the fourth, fifth, and sixth rings?

- There is 1 hexagon in a honeycomb with 1 ring. There are 7 hexagons in a honeycomb with 2 rings. How many hexagons are there in honeycombs with 3 rings, 4 rings, 5 rings, and 6 rings?

Reading the Lesson

1. Give the degree and leading coefficient of each polynomial in one variable.

	degree	**leading coefficient**
a. $10x^3 + 3x^2 - x + 7$	________	________
b. $7y^2 - 2y^5 + y - 4y^3$	________	________
c. 100	________	________

2. Match each description of a polynomial function from the list on the left with the corresponding end behavior from the list on the right.

a. even degree, negative leading coefficient	**i.** $f(x) \to +\infty$ as $x \to +\infty$; $f(x) \to +\infty$ as $x \to -\infty$
b. odd degree, positive leading coefficient	**ii.** $f(x) \to -\infty$ as $x \to +\infty$; $f(x) \to +\infty$ as $x \to -\infty$
c. odd degree, negative leading coefficient	**iii.** $f(x) \to -\infty$ as $x \to +\infty$; $f(x) \to -\infty$ as $x \to -\infty$
d. even degree, positive leading coefficient	**iv.** $f(x) \to +\infty$ as $x \to +\infty$; $f(x) \to -\infty$ as $x \to -\infty$

Helping You Remember

3. What is an easy way to remember the difference between the end behavior of the graphs of even-degree and odd-degree polynomial functions?

7-1 Enrichment

Approximation by Means of Polynomials

Many scientific experiments produce pairs of numbers $[x, f(x)]$ that can be related by a formula. If the pairs form a function, you can fit a polynomial to the pairs in exactly one way. Consider the pairs given by the following table.

x	1	2	4	7
$f(x)$	6	11	39	−54

We will assume the polynomial is of degree three. Substitute the given values into this expression.

$$f(x) = A + B(x - x_0) + C(x - x_0)(x - x_1) + D(x - x_0)(x - x_1)(x - x_2)$$

You will get the system of equations shown below. You can solve this system and use the values for A, B, C, and D to find the desired polynomial.

$$6 = A$$
$$11 = A + B(2 - 1) = A + B$$
$$39 = A + B(4 - 1) + C(4 - 1)(4 - 2) = A + 3B + 6C$$
$$-54 = A + B(7 - 1) + C(7 - 1)(7 - 2) + D(7 - 1)(7 - 2)(7 - 4) = A + 6B + 30C + 90D$$

Solve.

1. Solve the system of equations for the values A, B, C, and D.

2. Find the polynomial that represents the four ordered pairs. Write your answer in the form $y = a + bx + cx^2 + dx^3$.

3. Find the polynomial that gives the following values.

x	8	12	15	20
$f(x)$	−207	169	976	3801

4. A scientist measured the volume $f(x)$ of carbon dioxide gas that can be absorbed by one cubic centimeter of charcoal at pressure x. Find the values for A, B, C, and D.

x	120	340	534	698
$f(x)$	3.1	5.5	7.1	8.3

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7-2 Study Guide and Intervention

Graphing Polynomial Functions

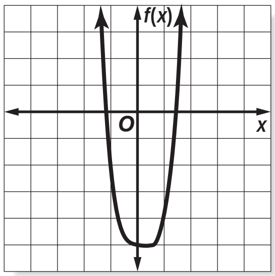
Graph Polynomial Functions

Location Principle	Suppose $y = f(x)$ represents a polynomial function and a and b are two numbers such that $f(a) < 0$ and $f(b) > 0$. Then the function has at least one real zero between a and b.

Example **Determine the values of x between which each real zero of the function $f(x) = 2x^4 - x^3 - 5$ is located. Then draw the graph.**

Make a table of values. Look at the values of $f(x)$ to locate the zeros. Then use the points to sketch a graph of the function.

x	$f(x)$
−2	35
−1	−2
0	−5
1	−4
2	19

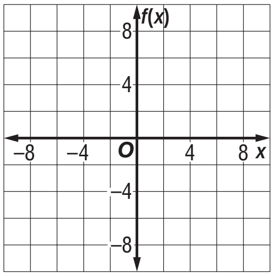


The changes in sign indicate that there are zeros between $x = -2$ and $x = -1$ and between $x = 1$ and $x = 2$.

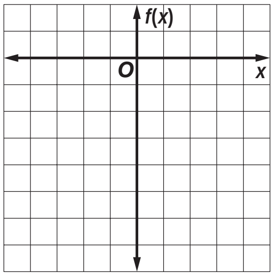
Exercises

Graph each function by making a table of values. Determine the values of x at which or between which each real zero is located.

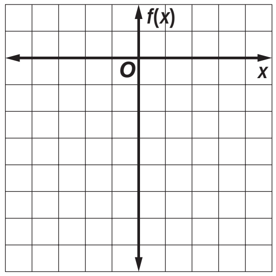
1. $f(x) = x^3 - 2x^2 + 1$



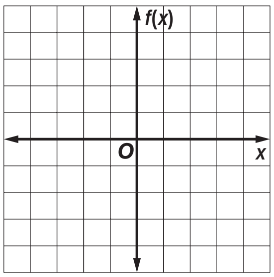
2. $f(x) = x^4 + 2x^3 - 5$



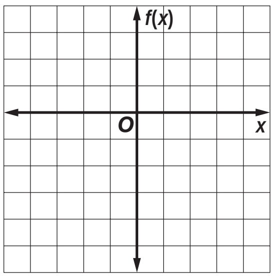
3. $f(x) = -x^4 + 2x^2 - 1$



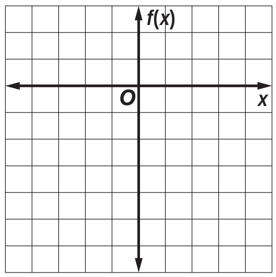
4. $f(x) = x^3 - 3x^2 + 4$



5. $f(x) = 3x^3 + 2x - 1$



6. $f(x) = x^4 - 3x^3 + 1$



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7-2 Study Guide and Intervention *(continued)*

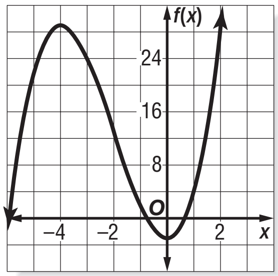
Graphing Polynomial Functions

Maximum and Minimum Points A quadratic function has either a maximum or a minimum point on its graph. For higher degree polynomial functions, you can find *turning points*, which represent **relative maximum** or **relative minimum** points.

Example **Graph $f(x) = x^3 + 6x^2 - 3$. Estimate the x-coordinates at which the relative maxima and minima occur.**

Make a table of values and graph the function.

x	$f(x)$	
−5	22	
−4	29	← indicates a relative maximum
−3	24	
−2	13	
−1	2	← zero between $x = -1$, $x = 0$
0	−3	← indicates a relative minimum
1	4	
2	29	

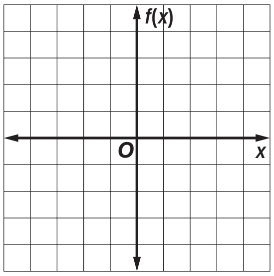


A relative maximum occurs at $x = -4$ and a relative minimum occurs at $x = 0$.

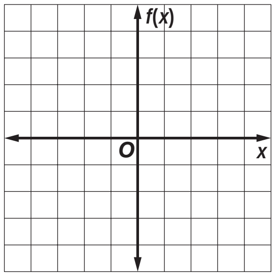
Exercises

Graph each function by making a table of values. Estimate the x-coordinates at which the relative maxima and minima occur.

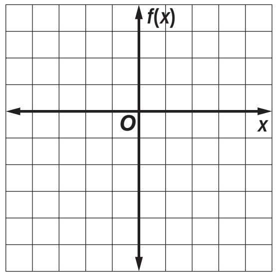
1. $f(x) = x^3 - 3x^2$



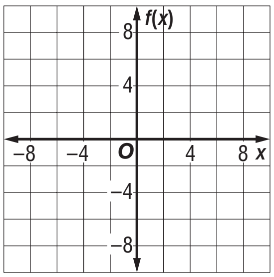
2. $f(x) = 2x^3 + x^2 - 3x$



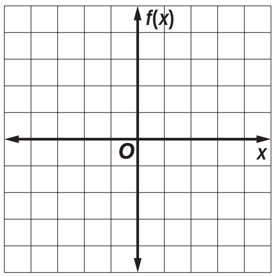
3. $f(x) = 2x^3 - 3x + 2$



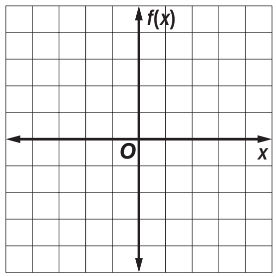
4. $f(x) = x^4 - 7x - 3$



5. $f(x) = x^5 - 2x^2 + 2$



6. $f(x) = x^3 + 2x^2 - 3$



NAME ______________________________ DATE ____________ PERIOD _____

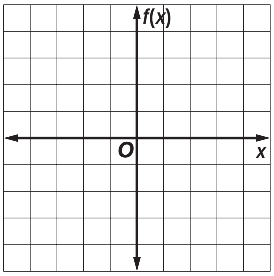
7-2 Skills Practice

Graphing Polynomial Functions

Complete each of the following.
a. Graph each function by making a table of values.
b. Determine consecutive values of *x* between which each real zero is located.
c. Estimate the *x*-coordinates at which the relative maxima and minima occur.

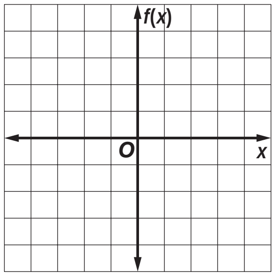
1. $f(x) = x^3 - 3x^2 + 1$

x	$f(x)$
−2	
−1	
0	
1	
2	
3	
4	



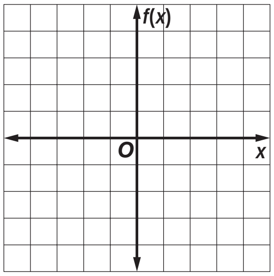
2. $f(x) = x^3 - 3x + 1$

x	$f(x)$
−3	
−2	
−1	
0	
1	
2	
3	



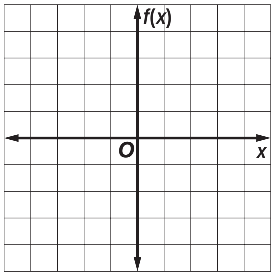
3. $f(x) = 2x^3 + 9x^2 + 12x + 2$

x	$f(x)$
−3	
−2	
−1	
0	
1	



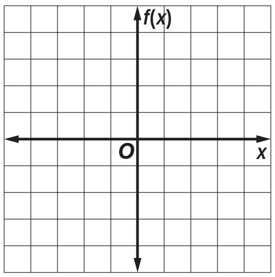
4. $f(x) = 2x^3 - 3x^2 + 2$

x	$f(x)$
−1	
0	
1	
2	
3	



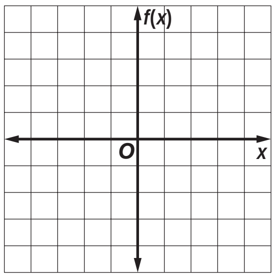
5. $f(x) = x^4 - 2x^2 - 2$

x	$f(x)$
−3	
−2	
−1	
0	
1	
2	
3	



6. $f(x) = 0.5x^4 - 4x^2 + 4$

x	$f(x)$
−3	
−2	
−1	
0	
1	
2	
3	



NAME ____ DATE ____________ PERIOD _____

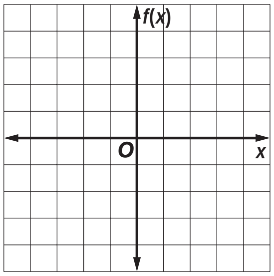
7-2 Practice

Graphing Polynomial Functions

Complete each of the following.
a. Graph each function by making a table of values.
b. Determine consecutive values of *x* between which each real zero is located.
c. Estimate the *x*-coordinates at which the relative and relative minima occur.

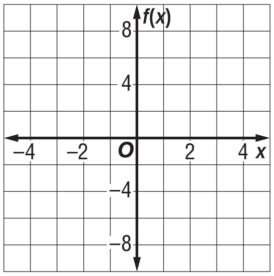
1. $f(x) = -x^3 + 3x^2 - 3$

x	*f*(*x*)
−2	
−1	
0	
1	
2	
3	
4	



2. $f(x) = x^3 - 1.5x^2 - 6x + 1$

x	*f*(*x*)
−2	
−1	
0	
1	
2	
3	
4	



3. $f(x) = 0.75x^4 + x^3 - 3x^2 + 4$

x	*f*(*x*)

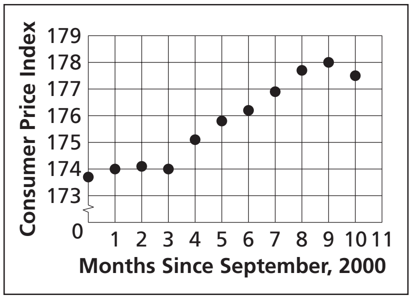
4. $f(x) = x^4 + 4x^3 + 6x^2 + 4x - 3$

x	*f*(*x*)

PRICES For Exercises 5 and 6, use the following information.

The Consumer Price Index (CPI) gives the relative price for a fixed set of goods and services. The CPI from September, 2000 to July, 2001 is shown in the graph.

Source: U. S. Bureau of Labor Statistics



5. Describe the turning points of the graph.

6. If the graph were modeled by a polynomial equation, what is the least degree the equation could have?

7. LABOR A town's jobless rate can be modeled by (1, 3.3), (2, 4.9), (3, 5.3), (4, 6.4), (5, 4.5), (6, 5.6), (7, 2.5), (8, 2.7). How many turning points would the graph of a polynomial function through these points have? Describe them.

NAME ____ DATE ____________ PERIOD _____

7-2 Reading to Learn Mathematics

Graphing Polynomial Functions

Pre-Activity **How can graphs of polynomial functions show trends in data?**

Read the introduction to Lesson 7-2 at the top of page 353 in your textbook.

Three points on the graph shown in your textbook are (0, 14), (70, 3.78), and (100, 9). Give the real-world meaning of the coordinates of these points.

Reading the Lesson

1. Suppose that $f(x)$ is a third-degree polynomial function and that c and d are real numbers, with $d > c$. Indicate whether each statement is *true* or *false*. (Remember that *true* means *always* true.)

a. If $f(c) > 0$ and $f(d) < 0$, there is exactly one real zero between c and d.

b. If $f(c) = f(d) \neq 0$, there are no real zeros between c and d.

c. If $f(c) < 0$ and $f(d) > 0$, there is at least one real zero between c and d.

2. Match each graph with its description.

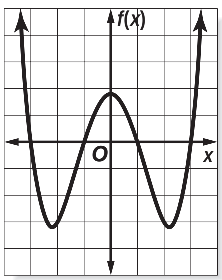
a. third-degree polynomial with one relative maximum and one relative minimum; leading coefficient negative

b. fourth-degree polynomial with two relative minima and one relative maximum

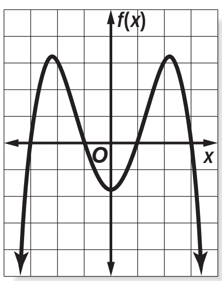
c. third-degree polynomial with one relative maximum and one relative minimum; leading coefficient positive

d. fourth-degree polynomial with two relative maxima and one relative minimum

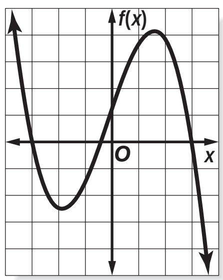
i.



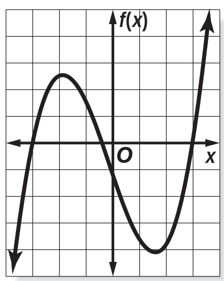
ii.



iii.



iv.



Helping You Remember

3. The origins of words can help you to remember their meaning and to distinguish between similar words. Look up *maximum* and *minimum* in a dictionary and describe their origins (original language and meaning).

Lesson 7-2

NAME ____ DATE ____________ PERIOD _____

7-2 Enrichment

Golden Rectangles

Use a straightedge, a compass, and the instructions below to construct a golden rectangle.

1. Construct square *ABCD* with sides of 2 centimeters.

2. Construct the midpoint of $\overline{AB}$. Call the midpoint *M*.

3. Using *M* as the center, set your compass opening at *MC*. Construct an arc with center *M* that intersects $\overline{AB}$. Call the point of intersection *P*.

4. Construct a line through *P* that is perpendicular to $\overline{AB}$.

5. Extend $\overline{DC}$ so that it intersects the perpendicular. Call the intersection point *Q*. *APQD* is a golden rectangle. Check this conclusion by finding the value of $\frac{QP}{AP}$.

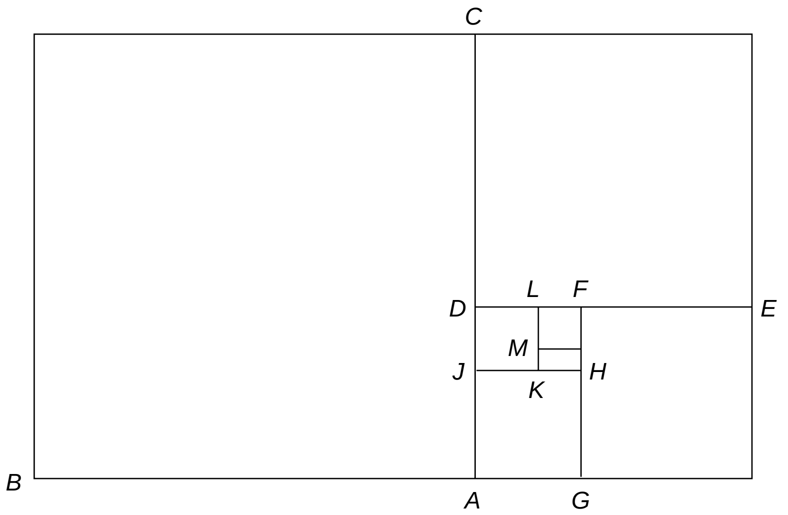
A figure consisting of similar golden rectangles is shown below. Use a compass and the instructions below to draw quarter-circle arcs that form a spiral like that found in the shell of a chambered nautilus.

6. Using *A* as a center, draw an arc that passes through *B* and *C*.

7. Using *D* as a center, draw an arc that passes through *C* and *E*.

8. Using *F* as a center, draw an arc that passes through *E* and *G*.

9. Continue drawing arcs, using *H*, *K*, and *M* as the centers.



7-3 Study Guide and Intervention

Solving Equations Using Quadratic Techniques

Quadratic Form Certain polynomial expressions in x can be written in the quadratic form $au^2 + bu + c$ for any numbers a, b, and c, $a \neq 0$, where u is an expression in x.

Example **Write each polynomial in quadratic form, if possible.**

a. $\mathbf{3a^6 - 9a^3 + 12}$

Let $u = a^3$.

$3a^6 - 9a^3 + 12 = 3(a^3)^2 - 9(a^3) + 12$

b. $\mathbf{101b - 49\sqrt{b} + 42}$

Let $u = \sqrt{b}$.

$101b - 49\sqrt{b} + 42 = 101(\sqrt{b})^2 - 49(\sqrt{b}) + 42$

c. $\mathbf{24a^5 + 12a^3 + 18}$

This expression cannot be written in quadratic form, since $a^5 \neq (a^3)^2$.

Exercises

Write each polynomial in quadratic form, if possible.

1. $x^4 + 6x^2 - 8$

2. $4p^4 + 6p^2 + 8$

3. $x^8 + 2x^4 + 1$

4. $x^{\frac{1}{8}} + 2x^{\frac{1}{16}} + 1$

5. $6x^4 + 3x^3 + 18$

6. $12x^4 + 10x^2 - 4$

7. $24x^8 + x^4 + 4$

8. $18x^6 - 2x^3 + 12$

9. $100x^4 - 9x^2 - 15$

10. $25x^8 + 36x^6 - 49$

11. $48x^6 - 32x^3 + 20$

12. $63x^8 + 5x^4 - 29$

13. $32x^{10} + 14x^5 - 143$

14. $50x^3 - 15x\sqrt{x} - 18$

15. $60x^6 - 7x^3 + 3$

16. $10x^{10} - 7x^5 - 7$

Lesson 7-3

NAME ______________________________ DATE __________ PERIOD _____

7-3 Study Guide and Intervention *(continued)*

Solving Equations Using Quadratic Techniques

Solve Equations Using Quadratic Form If a polynomial expression can be written in quadratic form, then you can use what you know about solving quadratic equations to solve the related polynomial equation.

Example 1 **Solve $x^4 - 40x^2 + 144 = 0$.**

$x^4 - 40x^2 + 144 = 0$	Original equation
$(x^2)^2 - 40(x^2) + 144 = 0$	Write the expression on the left in quadratic form.
$(x^2 - 4)(x^2 - 36) = 0$	Factor.
$x^2 - 4 = 0$ or $x^2 - 36 = 0$	Zero Product Property
$(x - 2)(x + 2) = 0$ or $(x - 6)(x + 6) = 0$	Factor.
$x - 2 = 0$ or $x + 2 = 0$ or $x - 6 = 0$ or $x + 6 = 0$	Zero Product Property
$x = 2$ or $x = -2$ or $x = 6$ or $x = -6$	Simplify.

The solutions are ± 2 and ± 6.

Example 2 **Solve $2x + \sqrt{x} - 15 = 0$.**

$2x + \sqrt{x} - 15 = 0$	Original equation
$2(\sqrt{x})^2 + \sqrt{x} - 15 = 0$	Write the expression on the left in quadratic form.
$(2\sqrt{x} - 5)(\sqrt{x} + 3) = 0$	Factor.
$2\sqrt{x} - 5 = 0$ or $\sqrt{x} + 3 = 0$	Zero Product Property
$\sqrt{x} = \frac{5}{2}$ or $\sqrt{x} = -3$	Simplify.

Since the principal square root of a number cannot be negative, $\sqrt{x} = -3$ has no solution.
The solution is $\frac{25}{4}$ or $6\frac{1}{4}$.

Exercises

Solve each equation.

1. $x^4 = 49$

2. $x^4 - 6x^2 = -8$

3. $x^4 - 3x^2 = 54$

4. $3t^6 - 48t^2 = 0$

5. $m^6 - 16m^3 + 64 = 0$

6. $y^4 - 5y^2 + 4 = 0$

7. $x^4 - 29x^2 + 100 = 0$

8. $4x^4 - 73x^2 + 144 = 0$

9. $\frac{1}{x^2} - \frac{7}{x} + 12 = 0$

10. $x - 5\sqrt{x} + 6 = 0$

11. $x - 10\sqrt{x} + 21 = 0$

12. $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$

NAME ______________________________ DATE __________ PERIOD _____

7-3

Skills Practice

Solving Equations Using Quadratic Techniques

Write each expression in quadratic form, if possible.

1. $5x^4 + 2x^2 - 8$

2. $3y^8 - 4y^2 + 3$

3. $100a^6 + a^3$

4. $x^8 + 4x^4 + 9$

5. $12x^4 - 7x^2$

6. $6b^5 + 3b^3 - 1$

7. $15v^6 - 8v^3 + 9$

8. $a^9 - 5a^5 + 7a$

Solve each equation.

9. $a^3 - 9a^2 + 14a = 0$

10. $x^3 = 3x^2$

11. $t^4 - 3t^3 - 40t^2 = 0$

12. $b^3 - 8b^2 + 16b = 0$

13. $m^4 = 4$

14. $w^3 - 6w = 0$

15. $m^4 - 18m^2 = -81$

16. $x^5 - 81x = 0$

17. $h^4 - 10h^2 = -9$

18. $a^4 - 9a^2 + 20 = 0$

19. $y^4 - 7y^2 + 12 = 0$

20. $v^4 - 12v^2 + 35 = 0$

21. $x^5 - 7x^3 + 6x = 0$

22. $c^{\frac{2}{3}} + 7c^{\frac{1}{3}} + 12 = 0$

23. $z - 5\sqrt{z} = -6$

24. $x - 30\sqrt{x} + 200 = 0$

Lesson 7-3

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7-3 Practice

Solving Equations Using Quadratic Techniques

Write each expression in quadratic form, if possible.

1. $10b^4 + 3b^2 - 11$

2. $-5x^8 + x^2 + 6$

3. $28d^6 + 25d^3$

4. $4s^8 + 4s^4 + 7$

5. $500x^4 - x^2$

6. $8b^5 - 8b^3 - 1$

7. $32w^5 - 56w^3 + 8w$

8. $e^{\frac{2}{3}} + 7e^{\frac{1}{3}} - 10$

9. $x^{\frac{1}{5}} + 29x^{\frac{1}{10}} + 2$

Solve each equation.

10. $y^4 - 7y^3 - 18y^2 = 0$

11. $s^5 + 4s^4 - 32s^3 = 0$

12. $m^4 - 625 = 0$

13. $n^4 - 49n^2 = 0$

14. $x^4 - 50x^2 + 49 = 0$

15. $t^4 - 21t^2 + 80 = 0$

16. $4r^6 - 9r^4 = 0$

17. $x^4 - 24 = -2x^2$

18. $d^4 = 16d^2 - 48$

19. $t^3 - 343 = 0$

20. $x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 6 = 0$

21. $x^{\frac{4}{3}} - 29x^{\frac{2}{3}} + 100 = 0$

22. $y^3 - 28y^{\frac{3}{2}} + 27 = 0$

23. $n - 10\sqrt{n} + 25 = 0$

24. $w - 12\sqrt{w} + 27 = 0$

25. $x - 2\sqrt{x} - 80 = 0$

26. **PHYSICS** A proton in a magnetic field follows a path on a coordinate grid modeled by the function $f(x) = x^4 - 2x^2 - 15$. What are the x-coordinates of the points on the grid where the proton crosses the x-axis?

27. **SURVEYING** Vista county is setting aside a large parcel of land to preserve it as open space. The county has hired Meghan's surveying firm to survey the parcel, which is in the shape of a right triangle. The longer leg of the triangle measures 5 miles less than the square of the shorter leg, and the hypotenuse of the triangle measures 13 miles less than twice the square of the shorter leg. The length of each boundary is a whole number. Find the length of each boundary.

7-3 Reading to Learn Mathematics

Solving Equations Using Quadratic Techniques

Pre-Activity **How can solving polynomial equations help you to find dimensions?**

Read the introduction to Lesson 7-3 at the top of page 360 in your textbook.

Explain how the formula given for the volume of the box can be obtained from the dimensions shown in the figure.

Reading the Lesson

1. Which of the following expressions can be written in quadratic form?

a. $x^3 + 6x^2 + 9$ **b.** $x^4 - 7x^2 + 6$ **c.** $m^6 + 4m^3 + 4$

d. $y - 2y^{\frac{1}{2}} - 15$ **e.** $x^5 + x^3 + 1$ **f.** $r^4 + 6 - r^8$

g. $p^{\frac{1}{4}} + 8p^{\frac{1}{2}} + 12$ **h.** $r^{\frac{1}{3}} + 2r^{\frac{1}{6}} - 3$ **i.** $5\sqrt{z} + 2z - 3$

2. Match each expression from the list on the left with its factorization from the list on the right.

a. $x^4 - 3x^2 - 40$ **i.** $(x^3 + 3)(x^3 - 3)$

b. $x^4 - 10x^2 + 25$ **ii.** $(\sqrt{x} + 3)(\sqrt{x} - 3)$

c. $x^6 - 9$ **iii.** $(\sqrt{x} + 3)^2$

d. $x - 9$ **iv.** $(x^2 + 1)(x^4 - x^2 + 1)$

e. $x^6 + 1$ **v.** $(x^2 - 5)^2$

f. $x + 6\sqrt{x} + 9$ **vi.** $(x^2 + 5)(x^2 - 8)$

Helping You Remember

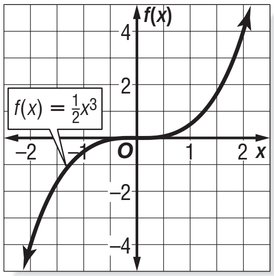
3. What is an easy way to tell whether a trinomial in one variable containing one constant term can be written in quadratic form?

Lesson 7-3

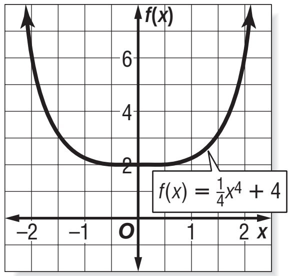
7-3 Enrichment

Odd and Even Polynomial Functions

Functions whose graphs are symmetric with respect to the origin are called *odd* functions. If $f(-x) = -f(x)$ for all x in the domain of $f(x)$, then $f(x)$ is odd.



Functions whose graphs are symmetric with respect to the y-axis are called *even* functions. If $f(-x) = f(x)$ for all x in the domain of $f(x)$, then $f(x)$ is even.



Example **Determine whether $f(x) = x^3 - 3x$ is odd, even, or neither.**

$f(x) = x^3 - 3x$

$f(-x) = (-x)^3 - 3(-x)$ Replace x with $-x$.

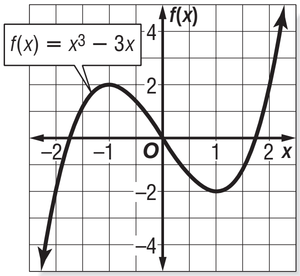
$= -x^3 + 3x$ Simplify.

$= -(x^3 - 3x)$ Factor out -1.

$= -f(x)$ Substutute.

Therefore, $f(x)$ is odd.

The graph at the right verifies that $f(x)$ is odd. The graph of the function is symmetric with respect to the origin.



Determine whether each function is *odd*, *even*, or *neither* by graphing or by applying the rules for odd and even functions.

1. $f(x) = 4x^2$

2. $f(x) = -7x^4$

3. $f(x) = x^7$

4. $f(x) = x^3 - x^2$

5. $f(x) = 3x^3 + 1$

6. $f(x) = x^8 - x^5 - 6$

7. $f(x) = -8x^5 - 2x^3 + 6x$

8. $f(x) = x^4 - 3x^3 + 2x^2 - 6x + 1$

9. $f(x) = x^4 + 3x^2 + 11$

10. $f(x) = x^7 - 6x^5 + 2x^3 + x$

11. Complete the following definitions: A polynomial function is odd if and only if all the terms are of _____________ degrees. A polynomial function is even if and only if all the terms are of _____________ degrees.

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7-4 Study Guide and Intervention

The Remainder and Factor Theorems

Synthetic Substitution

Remainder Theorem	The remainder, when you divide the polynomial $f(x)$ by $(x - a)$, is the constant $f(a)$. $f(x) = q(x) \cdot (x - a) + f(a)$, where $q(x)$ is a polynomial with degree one less than the degree of $f(x)$.

Example 1 **If $f(x) = 3x^4 + 2x^3 - 5x^2 + x - 2$, find $f(-2)$.**

Method 1 Synthetic Substitution

By the Remainder Theorem, $f(-2)$ should be the remainder when you divide the polynomial by $x + 2$.

−2	3	2	−5	1	−2
		−6	8	−6	10
	3	−4	3	−5	8

The remainder is 8, so $f(-2) = 8$.

Method 2 Direct Substitution

Replace x with -2.

$f(x) = 3x^4 + 2x^3 - 5x^2 + x - 2$

$f(-2) = 3(-2)^4 + 2(-2)^3 - 5(-2)^2 + (-2) - 2$

$= 48 - 16 - 20 - 2 - 2$ or 8

So $f(-2) = 8$.

Example 2 **If $f(x) = 5x^3 + 2x - 1$, find $f(3)$.**

Again, by the Remainder Theorem, $f(3)$ should be the remainder when you divide the polynomial by $x - 3$.

3	5	0	2	−1
		15	45	141
	5	15	47	140

The remainder is 140, so $f(3) = 140$.

Exercises

Use synthetic substitution to find $f(-5)$ and $f\left(\frac{1}{2}\right)$ for each function.

1. $f(x) = -3x^2 + 5x - 1$

2. $f(x) = 4x^2 + 6x - 7$

3. $f(x) = -x^3 + 3x^2 - 5$

4. $f(x) = x^4 + 11x^2 - 1$

Use synthetic substitution to find $f(4)$ and $f(-3)$ for each function.

5. $f(x) = 2x^3 + x^2 - 5x + 3$

6. $f(x) = 3x^3 - 4x + 2$

7. $f(x) = 5x^3 - 4x^2 + 2$

8. $f(x) = 2x^4 - 4x^3 + 3x^2 + x - 6$

9. $f(x) = 5x^4 + 3x^3 - 4x^2 - 2x + 4$

10. $f(x) = 3x^4 - 2x^3 - x^2 + 2x - 5$

11. $f(x) = 2x^4 - 4x^3 - x^2 - 6x + 3$

12. $f(x) = 4x^4 - 4x^3 + 3x^2 - 2x - 3$

7-4 Study Guide and Intervention *(continued)*

The Remainder and Factor Theorems

Factors of Polynomials The **Factor Theorem** can help you find all the factors of a polynomial.

Factor Theorem	The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.

Example **Show that $x + 5$ is a factor of $x^3 + 2x^2 - 13x + 10$. Then find the remaining factors of the polynomial.**

By the Factor Theorem, the binomial $x + 5$ is a factor of the polynomial if -5 is a zero of the polynomial function. To check this, use synthetic substitution.

$$\begin{array}{r|rrrr} -5 & 1 & 2 & -13 & 10 \\ & & -5 & 15 & -10 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

Since the remainder is 0, $x + 5$ is a factor of the polynomial. The polynomial $x^3 + 2x^2 - 13x + 10$ can be factored as $(x + 5)(x^2 - 3x + 2)$. The depressed polynomial $x^2 - 3x + 2$ can be factored as $(x - 2)(x - 1)$.

So $x^3 + 2x^2 - 13x + 10 = (x + 5)(x - 2)(x - 1)$.

Exercises

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

1. $x^3 + x^2 - 10x + 8$; $x - 2$

2. $x^3 - 4x^2 - 11x + 30$; $x + 3$

3. $x^3 + 15x^2 + 71x + 105$; $x + 7$

4. $x^3 - 7x^2 - 26x + 72$; $x + 4$

5. $2x^3 - x^2 - 7x + 6$; $x - 1$

6. $3x^3 - x^2 - 62x - 40$; $x + 4$

7. $12x^3 - 71x^2 + 57x - 10$; $x - 5$

8. $14x^3 + x^2 - 24x + 9$; $x - 1$

9. $x^3 + x + 10$; $x + 2$

10. $2x^3 - 11x^2 + 19x - 28$; $x - 4$

11. $3x^3 - 13x^2 - 34x + 24$; $x - 6$

12. $x^4 + x^3 - 11x^2 - 9x + 18$; $x - 1$

NAME ____ DATE ____________ PERIOD _____

7-4 Skills Practice

The Remainder and Factor Theorems

Use synthetic substitution to find $f(2)$ and $f(-1)$ for each function.

1. $f(x) = x^2 + 6x + 5$

2. $f(x) = x^2 - x + 1$

3. $f(x) = x^2 - 2x - 2$

4. $f(x) = x^3 + 2x^2 + 5$

5. $f(x) = x^3 - x^2 - 2x + 3$

6. $f(x) = x^3 + 6x^2 + x - 4$

7. $f(x) = x^3 - 3x^2 + x - 2$

8. $f(x) = x^3 - 5x^2 - x + 6$

9. $f(x) = x^4 + 2x^2 - 9$

10. $f(x) = x^4 - 3x^3 + 2x^2 - 2x + 6$

11. $f(x) = x^5 - 7x^3 - 4x + 10$

12. $f(x) = x^6 - 2x^5 + x^4 + x^3 - 9x^2 - 20$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

13. $x^3 + 2x^2 - x - 2;\ x + 1$

14. $x^3 + x^2 - 5x + 3;\ x - 1$

15. $x^3 + 3x^2 - 4x - 12;\ x + 3$

16. $x^3 - 6x^2 + 11x - 6;\ x - 3$

17. $x^3 + 2x^2 - 33x - 90;\ x + 5$

18. $x^3 - 6x^2 + 32;\ x - 4$

19. $x^3 - x^2 - 10x - 8;\ x + 2$

20. $x^3 - 19x + 30;\ x - 2$

21. $2x^3 + x^2 - 2x - 1;\ x + 1$

22. $2x^3 + x^2 - 5x + 2;\ x + 2$

23. $3x^3 + 4x^2 - 5x - 2;\ 3x + 1$

24. $3x^3 + x^2 + x - 2;\ 3x - 2$

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7-4 Practice

The Remainder and Factor Theorems

Use synthetic substitution to find $f(-3)$ and $f(4)$ for each function.

1. $f(x) = x^2 + 2x + 3$

2. $f(x) = x^2 - 5x + 10$

3. $f(x) = x^2 - 5x - 4$

4. $f(x) = x^3 - x^2 - 2x + 3$

5. $f(x) = x^3 + 2x^2 + 5$

6. $f(x) = x^3 - 6x^2 + 2x$

7. $f(x) = x^3 - 2x^2 - 2x + 8$

8. $f(x) = x^3 - x^2 + 4x - 4$

9. $f(x) = x^3 + 3x^2 + 2x - 50$

10. $f(x) = x^4 + x^3 - 3x^2 - x + 12$

11. $f(x) = x^4 - 2x^2 - x + 7$

12. $f(x) = 2x^4 - 3x^3 + 4x^2 - 2x + 1$

13. $f(x) = 2x^4 - x^3 + 2x^2 - 26$

14. $f(x) = 3x^4 - 4x^3 + 3x^2 - 5x - 3$

15. $f(x) = x^5 + 7x^3 - 4x - 10$

16. $f(x) = x^6 + 2x^5 - x^4 + x^3 - 9x^2 + 20$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

17. $x^3 + 3x^2 - 6x - 8$; $x - 2$

18. $x^3 + 7x^2 + 7x - 15$; $x - 1$

19. $x^3 - 9x^2 + 27x - 27$; $x - 3$

20. $x^3 - x^2 - 8x + 12$; $x + 3$

21. $x^3 + 5x^2 - 2x - 24$; $x - 2$

22. $x^3 - x^2 - 14x + 24$; $x + 4$

23. $3x^3 - 4x^2 - 17x + 6$; $x + 2$

24. $4x^3 - 12x^2 - x + 3$; $x - 3$

25. $18x^3 + 9x^2 - 2x - 1$; $2x + 1$

26. $6x^3 + 5x^2 - 3x - 2$; $3x - 2$

27. $x^5 + x^4 - 5x^3 - 5x^2 + 4x + 4$; $x + 1$

28. $x^5 - 2x^4 + 4x^3 - 8x^2 - 5x + 10$; $x - 2$

29. POPULATION The projected population in thousands for a city over the next several years can be estimated by the function $P(x) = x^3 + 2x^2 - 8x + 520$, where x is the number of years since 2000. Use synthetic substitution to estimate the population for 2005.

30. VOLUME The volume of water in a rectangular swimming pool can be modeled by the polynomial $2x^3 - 9x^2 + 7x + 6$. If the depth of the pool is given by the polynomial $2x + 1$, what polynomials express the length and width of the pool?

NAME ______________________________ DATE ____________ PERIOD _____

7-4 Reading to Learn Mathematics

The Remainder and Factor Theorems

Pre-Activity **How can you use the Remainder Theorem to evaluate polynomials?**

Read the introduction to Lesson 7-4 at the top of page 365 in your textbook.

Show how you would use the model in the introduction to estimate the number of international travelers (in millions) to the United States in the year 2000. (Show how you would substitute numbers, but do not actually calculate the result.)

Reading the Lesson

1. Consider the following synthetic division.

1	3	2	−6	4
		3	5	−1
	3	5	−1	3

a. Using the division symbol ÷, write the division problem that is represented by this synthetic division. (Do not include the answer.)

b. Identify each of the following for this division.

dividend ______________________ divisor ______________________

quotient ______________________ remainder ______________________

c. If $f(x) = 3x^3 + 2x^2 - 6x + 4$, what is $f(1)$?

2. Consider the following synthetic division.

−3	1	0	0	27
		−3	9	−27
	1	−3	9	0

a. This division shows that ____________ is a factor of ________________.

b. The division shows that ________ is a zero of the polynomial function $f(x) =$ ________________.

c. The division shows that the point ____________ is on the graph of the polynomial function $f(x) =$ ________________.

Lesson 7-4

Helping You Remember

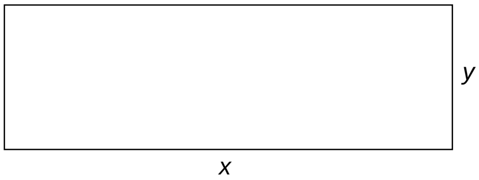
3. Think of a mnemonic for remembering the sentence, "Dividend equals quotient times divisor plus remainder."

7-4 Enrichment

Using Maximum Values

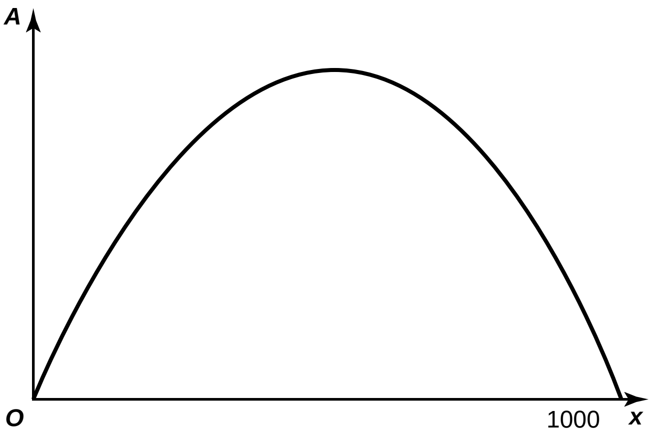
Many times maximum solutions are needed for different situations. For instance, what is the area of the largest rectangular field that can be enclosed with 2000 feet of fencing?

Let x and y denote the length and width of the field, respectively.



Perimeter: $2x + 2y = 2000 \rightarrow y = 1000 - x$
Area: $A = xy = x(1000 - x) = -x^2 + 1000x$

This problem is equivalent to finding the highest point on the graph of $A(x) = -x^2 + 1000x$ shown on the right.



Complete the square for $-x^2 + 1000x$.

$$A = -(x^2 - 1000x + 500^2) + 500^2$$
$$= -(x - 500)^2 + 500^2$$

Because the term $-(x - 500)^2$ is either negative or 0, the greatest value of A is 500^2. The maximum area enclosed is 500^2 or 250,000 square feet.

Solve each problem.

1. Find the area of the largest rectangular garden that can be enclosed by 300 feet of fence.

2. A farmer will make a rectangular pen with 100 feet of fence using part of his barn for one side of the pen. What is the largest area he can enclose?

3. An area along a straight stone wall is to be fenced. There are 600 meters of fencing available. What is the greatest rectangular area that can be enclosed?

7-5 Study Guide and Intervention

Roots and Zeros

Types of Roots The following statements are equivalent for any polynomial function $f(x)$.

- c is a zero of the polynomial function $f(x)$.
- $(x - c)$ is a factor of the polynomial $f(x)$.
- c is a root or solution of the polynomial equation $f(x) = 0$.

If c is real, then $(c, 0)$ is an intercept of the graph of $f(x)$.

Fundamental Theorem of Algebra	Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.
Corollary to the Fundamental Theorem of Algebras	A polynomial equation of the form $P(x) = 0$ of degree n with complex coefficients has exactly n roots in the set of complex numbers.
Descartes' Rule of Signs	If $P(x)$ is a polynomial with real coefficients whose terms are arranged in descending powers of the variable, • the number of positive real zeros of $y = P(x)$ is the same as the number of changes in sign of the coefficients of the terms, or is less than this by an even number, and • the number of negative real zeros of $y = P(x)$ is the same as the number of changes in sign of the coefficients of the terms of $P(-x)$, or is less than this number by an even number.

Example 1 **Solve the equation $6x^3 + 3x = 0$ and state the number and type of roots.**

$6x^3 + 3x = 0$

$3x(2x^2 + 1) = 0$

Use the Zero Product Property.

$3x = 0$ or $2x^2 + 1 = 0$

$x = 0$ or $2x^2 = -1$

$x = \pm\frac{i\sqrt{2}}{2}$

The equation has one real root, 0, and two imaginary roots, $\pm\frac{i\sqrt{2}}{2}$.

Example 2 **State the number of positive real zeros, negative real zeros, and imaginary zeros for $p(x) = 4x^4 - 3x^3 + x^2 + 2x - 5$.**

Since $p(x)$ has degree 4, it has 4 zeros.

Use Descartes' Rule of Signs to determine the number and type of real zeros. Since there are three sign changes, there are 3 or 1 positive real zeros.

Find $p(-x)$ and count the number of changes in sign for its coefficients.

$p(-x) = 4(-x)^4 - 3(-x)^3 + (-x)^2 + 2(-x) - 5$

$= 4x^4 + 3x^3 + x^2 - 2x - 5$

Since there is one sign change, there is exactly 1 negative real zero.

Exercises

Solve each equation and state the number and type of roots.

1. $x^2 + 4x - 21 = 0$

2. $2x^3 - 50x = 0$

3. $12x^3 + 100x = 0$

State the number of positive real zeros, negative real zeros, and imaginary zeros for each function.

4. $f(x) = 3x^3 + x^2 - 8x - 12$

5. $f(x) = 2x^4 - x^3 - 3x + 7$

Lesson 7-5

7-5 Study Guide and Intervention *(continued)*

Roots and Zeros

Find Zeros

Complex Conjugate Theorem	Suppose a and b are real numbers with $b \neq 0$. If $a + bi$ is a zero of a polynomial function with real coefficients, then $a - bi$ is also a zero of the function.

Example **Find all of the zeros of $f(x) = x^4 - 15x^2 + 38x - 60$.**

Since $f(x)$ has degree 4, the function has 4 zeros.

$f(x) = x^4 - 15x^2 + 38x - 60 \qquad f(-x) = x^4 - 15x^2 - 38x - 60$

Since there are 3 sign changes for the coefficients of $f(x)$, the function has 3 or 1 positive real zeros. Since there is 1 sign change for the coefficients of $f(-x)$, the function has 1 negative real zero. Use synthetic substitution to test some possible zeros.

2	1	0	−15	38	−60
		2	4	−22	32
	1	2	−11	16	−28

3	1	0	−15	38	−60
		3	9	−18	60
	1	3	−6	20	0

So 3 is a zero of the polynomial function. Now try synthetic substitution again to find a zero of the depressed polynomial.

−2	1	3	−6	20
		−2	−2	16
	1	1	−8	36

−4	1	3	−6	20
		−4	4	8
	1	−1	−2	28

−5	1	3	−6	20
		−5	10	−20
	1	−2	4	0

So -5 is another zero. Use the Quadratic Formula on the depressed polynomial $x^2 - 2x + 4$ to find the other 2 zeros, $1 \pm i\sqrt{3}$.

The function has two real zeros at 3 and -5 and two imaginary zeros at $1 \pm i\sqrt{3}$.

Exercises

Find all of the zeros of each function.

1. $f(x) = x^3 + x^2 + 9x + 9$

2. $f(x) = x^3 - 3x^2 + 4x - 12$

3. $p(a) = a^3 - 10a^2 + 34a - 40$

4. $p(x) = x^3 - 5x^2 + 11x - 15$

5. $f(x) = x^3 + 6x + 20$

6. $f(x) = x^4 - 3x^3 + 21x^2 - 75x - 100$

7-5 Skills Practice

Roots and Zeros

Solve each equation. State the number and type of roots.

1. $5x + 12 = 0$

2. $x^2 - 4x + 40 = 0$

3. $x^5 + 4x^3 = 0$

4. $x^4 + 625 = 0$

5. $4x^2 - 4x - 1 = 0$

6. $x^5 - 81x = 0$

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

7. $g(x) = 3x^3 - 4x^2 - 17x + 6$

8. $h(x) = 4x^3 - 12x^2 - x + 3$

9. $f(x) = x^3 - 8x^2 + 2x - 4$

10. $p(x) = x^3 - x^2 + 4x - 6$

11. $q(x) = x^4 + 7x^2 + 3x - 9$

12. $f(x) = x^4 - x^3 - 5x^2 + 6x + 1$

Find all the zeros of each function.

13. $h(x) = x^3 - 5x^2 + 5x + 3$

14. $g(x) = x^3 - 6x^2 + 13x - 10$

15. $h(x) = x^3 + 4x^2 + x - 6$

16. $q(x) = x^3 + 3x^2 - 6x - 8$

17. $g(x) = x^4 - 3x^3 - 5x^2 + 3x + 4$

18. $f(x) = x^4 - 21x^2 + 80$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

19. $-3, -5, 1$

20. $3i$

21. $-5 + i$

22. $-1, \sqrt{3}, -\sqrt{3}$

23. $i, 5i$

24. $-1, 1, i\sqrt{6}$

7-5 Practice

Roots and Zeros

Solve each equation. State the number and type of roots.

1. $-9x - 15 = 0$

2. $x^4 - 5x^2 + 4 = 0$

3. $x^5 = 81x$

4. $x^3 + x^2 - 3x - 3 = 0$

5. $x^3 + 6x + 20 = 0$

6. $x^4 - x^3 - x^2 - x - 2 = 0$

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

7. $f(x) = 4x^3 - 2x^2 + x + 3$

8. $p(x) = 2x^4 - 2x^3 + 2x^2 - x - 1$

9. $q(x) = 3x^4 + x^3 - 3x^2 + 7x + 5$

10. $h(x) = 7x^4 + 3x^3 - 2x^2 - x + 1$

Find all the zeros of each function.

11. $h(x) = 2x^3 + 3x^2 - 65x + 84$

12. $p(x) = x^3 - 3x^2 + 9x - 7$

13. $h(x) = x^3 - 7x^2 + 17x - 15$

14. $q(x) = x^4 + 50x^2 + 49$

15. $g(x) = x^4 + 4x^3 - 3x^2 - 14x - 8$

16. $f(x) = x^4 - 6x^3 + 6x^2 + 24x - 40$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

17. $-5, 3\boldsymbol{i}$

18. $-2, 3 + \boldsymbol{i}$

19. $-1, 4, 3\boldsymbol{i}$

20. $2, 5, 1 + \boldsymbol{i}$

21. CRAFTS Stephan has a set of plans to build a wooden box. He wants to reduce the volume of the box to 105 cubic inches. He would like to reduce the length of each dimension in the plan by the same amount. The plans call for the box to be 10 inches by 8 inches by 6 inches. Write and solve a polynomial equation to find out how much Stephen should take from each dimension.

7-5 Reading to Learn Mathematics

Roots and Zeros

Pre-Activity **How can the roots of an equation be used in pharmacology?**

Read the introduction to Lesson 7-5 at the top of page 371 in your textbook.

Using the model given in the introduction, write a polynomial equation with 0 on one side that can be solved to find the time or times at which there is 100 milligrams of medication in a patient's bloodstream.

Reading the Lesson

1. Indicate whether each statement is *true* or *false*.

a. Every polynomial equation of degree greater than one has at least one root in the set of real numbers.

b. If c is a root of the polynomial equation $f(x) = 0$, then $(x - c)$ is a factor of the polynomial $f(x)$.

c. If $(x + c)$ is a factor of the polynomial $f(x)$, then c is a zero of the polynomial function f.

d. A polynomial function f of degree n has exactly $(n - 1)$ complex zeros.

2. Let $f(x) = x^6 - 2x^5 + 3x^4 - 4x^3 + 5x^2 + 6x - 7$.

a. What are the possible numbers of positive real zeros of f?

b. Write $f(-x)$ in simplified form (with no parentheses).

What are the possible numbers of negative real zeros of f?

c. Complete the following chart to show the possible combinations of positive real zeros, negative real zeros, and imaginary zeros of the polynomial function f.

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros	Total Number of Zeros

Helping You Remember

3. It is easier to remember mathematical concepts and results if you relate them to each other. How can the Complex Conjugates Theorem help you remember the part of Descartes' Rule of Signs that says, "or is less than this number by an even number."

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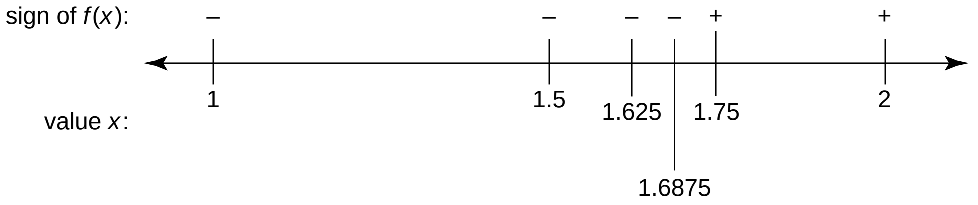
7-5 Enrichment

The Bisection Method for Approximating Real Zeros

The **bisection method** can be used to approximate zeros of polynomial functions like $f(x) = x^3 + x^2 - 3x - 3$.

Since $f(1) = -4$ and $f(2) = 3$, there is at least one real zero between 1 and 2. The midpoint of this interval is $\frac{1 + 2}{2} = 1.5$. Since $f(1.5) = -1.875$, the zero is between 1.5 and 2. The midpoint of this interval is $\frac{1.5 + 2}{2} = 1.75$. Since $f(1.75)$ is about 0.172, the zero is between 1.5 and 1.75. The midpoint of this interval is $\frac{1.5 + 1.75}{2} = 1.625$ and $f(1.625)$ is about -0.94. The zero is between 1.625 and 1.75. The midpoint of this interval is $\frac{1.625 + 1.75}{2} = 1.6875$. Since $f(1.6875)$ is about -0.41, the zero is between 1.6875 and 1.75. Therefore, the zero is 1.7 to the nearest tenth.

The diagram below summarizes the results obtained by the bisection method.



Using the bisection method, approximate to the nearest tenth the zero between the two integral values of x for each function.

1. $f(x) = x^3 - 4x^2 - 11x + 2, f(0) = 2, f(1) = -12$

2. $f(x) = 2x^4 + x^2 - 15, f(1) = -12, f(2) = 21$

3. $f(x) = x^5 - 2x^3 - 12, f(1) = -13, f(2) = 4$

4. $f(x) = 4x^3 - 2x + 7, f(-2) = -21, f(-1) = 5$

5. $f(x) = 3x^3 - 14x^2 - 27x + 126, f(4) = -14, f(5) = 16$

7-6 Study Guide and Intervention

Rational Zero Theorem

Identify Rational Zeros

Rational Zero Theorem	Let $f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-2}x^2 + a_{n-1}x + a^n$ represent a polynomial function with integral coefficients. If $\frac{p}{q}$ is a rational number in simplest form and is a zero of $y = f(x)$, then p is a factor of a_n and q is a factor of a_0.
Corollary (Integral Zero Theorem)	If the coefficients of a polynomial are integers such that $a_0 = 1$ and $a_n \neq 0$, any rational zeros of the function must be factors of a_n.

Example **List all of the possible rational zeros of each function.**

a. $f(x) = 3x^4 - 2x^2 + 6x - 10$

If $\frac{p}{q}$ is a rational root, then p is a factor of -10 and q is a factor of 3. The possible values for p are ±1, ±2, ±5, and ±10. The possible values for q are ±1 and ±3. So all of the possible rational zeros are $\frac{p}{q} = \pm1, \pm2, \pm5, \pm10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}$, and $\pm\frac{10}{3}$.

b. $q(x) = x^3 - 10x^2 + 14x - 36$

Since the coefficient of x^3 is 1, the possible rational zeros must be the factors of the constant term -36. So the possible rational zeros are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm9, \pm12, \pm18$, and ±36.

Exercises

List all of the possible rational zeros of each function.

1. $f(x) = x^3 + 3x^2 - x + 8$

2. $g(x) = x^5 - 7x^4 + 3x^2 + x - 20$

3. $h(x) = x^4 - 7x^3 - 4x^2 + x - 49$

4. $p(x) = 2x^4 - 5x^3 + 8x^2 + 3x - 5$

5. $q(x) = 3x^4 - 5x^3 + 10x + 12$

6. $r(x) = 4x^5 - 2x + 18$

7. $f(x) = x^7 - 6x^5 - 3x^4 + x^3 + 4x^2 - 120$

8. $g(x) = 5x^6 - 3x^4 + 5x^3 + 2x^2 - 15$

9. $h(x) = 6x^5 - 3x^4 + 12x^3 + 18x^2 - 9x + 21$

10. $p(x) = 2x^7 - 3x^6 + 11x^5 - 20x^2 + 11$

7-6 Study Guide and Intervention *(continued)*

Rational Zero Theorem

Find Rational Zeros

Example 1 **Find all of the rational zeros of $f(x) = 5x^3 + 12x^2 - 29x + 12$.**

From the corollary to the Fundamental Theorem of Algebra, we know that there are exactly 3 complex roots. According to Descartes' Rule of Signs there are 2 or 0 positive real roots and 1 negative real root. The possible rational zeros are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{4}{5}, \pm\frac{6}{5}, \pm\frac{12}{5}$. Make a table and test some possible rational zeros.

$\frac{p}{q}$	5	12	−29	12
1	5	17	−12	0

Since $f(1) = 0$, you know that $x = 1$ is a zero.

The depressed polynomial is $5x^2 + 17x - 12$, which can be factored as $(5x - 3)(x + 4)$.

By the Zero Product Property, this expression equals 0 when $x = \frac{3}{5}$ or $x = -4$.

The rational zeros of this function are 1, $\frac{3}{5}$, and -4.

Example 2 **Find all of the zeros of $f(x) = 8x^4 + 2x^3 + 5x^2 + 2x - 3$.**

There are 4 complex roots, with 1 positive real root and 3 or 1 negative real roots. The possible rational zeros are $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{8}, \pm\frac{3}{2}, \pm\frac{3}{4}$, and $\pm\frac{3}{8}$.

Make a table and test some possible values.

$\frac{p}{q}$	8	2	5	2	−3
1	8	10	15	17	14
2	8	18	41	84	165
$\frac{1}{2}$	8	6	8	6	0

Since $f\left(\frac{1}{2}\right) = 0$, we know that $x = \frac{1}{2}$ is a root.

The depressed polynomial is $8x^3 + 6x^2 + 8x + 6$.

Try synthetic substitution again. Any remaining rational roots must be negative.

$\frac{p}{q}$	8	6	8	6
$-\frac{1}{4}$	8	4	7	$4\frac{1}{4}$
$-\frac{3}{4}$	8	0	8	0

$x = -\frac{3}{4}$ is another rational root.

The depressed polynomial is $8x^2 + 8 = 0$, which has roots $\pm i$.

The zeros of this function are $\frac{1}{2}, -\frac{3}{4}$, and $\pm i$.

Exercises

Find all of the rational zeros of each function.

1. $f(x) = x^3 + 4x^2 - 25x - 28$

2. $f(x) = x^3 + 6x^2 + 4x + 24$

Find all of the zeros of each function.

3. $f(x) = x^4 + 2x^3 - 11x^2 + 8x - 60$

4. $f(x) = 4x^4 + 5x^3 + 30x^2 + 45x - 54$

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7-6 Skills Practice

Rational Zero Theorem

Lesson 7-6

List all of the possible rational zeros of each function.

1. $n(x) = x^2 + 5x + 3$

2. $h(x) = x^2 - 2x - 5$

3. $w(x) = x^2 - 5x + 12$

4. $f(x) = 2x^2 + 5x + 3$

5. $q(x) = 6x^3 + x^2 - x + 2$

6. $g(x) = 9x^4 + 3x^3 + 3x^2 - x + 27$

Find all of the rational zeros of each function.

7. $f(x) = x^3 - 2x^2 + 5x - 4 = 0$

8. $g(x) = x^3 - 3x^2 - 4x + 12$

9. $p(x) = x^3 - x^2 + x - 1$

10. $z(x) = x^3 - 4x^2 + 6x - 4$

11. $h(x) = x^3 - x^2 + 4x - 4$

12. $g(x) = 3x^3 - 9x^2 - 10x - 8$

13. $g(x) = 2x^3 + 7x^2 - 7x - 12$

14. $h(x) = 2x^3 - 5x^2 - 4x + 3$

15. $p(x) = 3x^3 - 5x^2 - 14x - 4 = 0$

16. $q(x) = 3x^3 + 2x^2 + 27x + 18$

17. $q(x) = 3x^3 - 7x^2 + 4$

18. $f(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$

19. $p(x) = x^4 - 5x^3 - 9x^2 - 25x - 70$

20. $n(x) = 16x^4 - 32x^3 - 13x^2 + 29x - 6$

Find all of the zeros of each function.

21. $f(x) = x^3 + 5x^2 + 11x + 15$

22. $q(x) = x^3 - 10x^2 + 18x - 4$

23. $m(x) = 6x^4 - 17x^3 + 8x^2 + 8x - 3$

24. $g(x) = x^4 + 4x^3 + 5x^2 + 4x + 4$

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7-6 Practice

Rational Zero Theorem

List all of the possible rational zeros of each function.

1. $h(x) = x^3 - 5x^2 + 2x + 12$

2. $s(x) = x^4 - 8x^3 + 7x - 14$

3. $f(x) = 3x^5 - 5x^2 + x + 6$

4. $p(x) = 3x^2 + x + 7$

5. $g(x) = 5x^3 + x^2 - x + 8$

6. $q(x) = 6x^5 + x^3 - 3$

Find all of the rational zeros of each function.

7. $q(x) = x^3 + 3x^2 - 6x - 8 = 0$

8. $v(x) = x^3 - 9x^2 + 27x - 27$

9. $c(x) = x^3 - x^2 - 8x + 12$

10. $f(x) = x^4 - 49x^2$

11. $h(x) = x^3 - 7x^2 + 17x - 15$

12. $b(x) = x^3 + 6x + 20$

13. $f(x) = x^3 - 6x^2 + 4x - 24$

14. $g(x) = 2x^3 + 3x^2 - 4x - 4$

15. $h(x) = 2x^3 - 7x^2 - 21x + 54 = 0$

16. $z(x) = x^4 - 3x^3 + 5x^2 - 27x - 36$

17. $d(x) = x^4 + x^3 + 16$

18. $n(x) = x^4 - 2x^3 - 3$

19. $p(x) = 2x^4 - 7x^3 + 4x^2 + 7x - 6$

20. $q(x) = 6x^4 + 29x^3 + 40x^2 + 7x - 12$

Find all of the zeros of each function.

21. $f(x) = 2x^4 + 7x^3 - 2x^2 - 19x - 12$

22. $q(x) = x^4 - 4x^3 + x^2 + 16x - 20$

23. $h(x) = x^6 - 8x^3$

24. $g(x) = x^6 - 1$

25. TRAVEL The height of a box that Joan is shipping is 3 inches less than the width of the box. The length is 2 inches more than twice the width. The volume of the box is 1540 in^3. What are the dimensions of the box?

26. GEOMETRY The height of a square pyramid is 3 meters shorter than the side of its base. If the volume of the pyramid is 432 m^3, how tall is it? Use the formula $V = \frac{1}{3}Bh$.

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7-6 Reading to Learn Mathematics

Rational Zero Theorem

Lesson 7-6

Pre-Activity **How can the Rational Zero Theorem solve problems involving large numbers?**

Read the introduction to Lesson 7-6 at the top of page 378 in your textbook.

Rewrite the polynomial equation $w(w + 8)(w - 5) = 2772$ in the form $f(x) = 0$, where $f(x)$ is a polynomial written in descending powers of x.

Reading the Lesson

1. For each of the following polynomial functions, list all the possible values of p, all the possible values of q, and all the possible rational zeros $\frac{p}{q}$.

a. $f(x) = x^3 - 2x^2 - 11x + 12$

possible values of p:

possible values of q:

possible values of $\frac{p}{q}$:

b. $f(x) = 2x^4 + 9x^3 - 23x^2 - 81x + 45$

possible values of p:

possible values of q:

possible values of $\frac{p}{q}$:

2. Explain in your own words how Descartes' Rule of Signs, the Rational Zero Theorem, and synthetic division can be used together to find all of the rational zeros of a polynomial function with integer coefficients.

Helping You Remember

3. Some students have trouble remembering which numbers go in the numerators and which go in the denominators when forming a list of possible rational zeros of a polynomial function. How can you use the linear polynomial equation $ax + b = 0$, where a and b are nonzero integers, to remember this?

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7-6 Enrichment

Infinite Continued Fractions

Some infinite expressions are actually equal to real numbers! The infinite continued fraction at the right is one example.

$$x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}$$

If you use x to stand for the infinite fraction, then the entire denominator of the first fraction on the right is also equal to x. This observation leads to the following equation:

$$x = 1 + \frac{1}{x}$$

Write a decimal for each continued fraction.

1. $1 + \frac{1}{1}$

2. $1 + \cfrac{1}{1 + \cfrac{1}{1}}$

3. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}$

4. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}}$

5. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}}}$

6. The more terms you add to the fractions above, the closer their value approaches the value of the infinite continued fraction. What value do the fractions seem to be approaching?

7. Rewrite $x = 1 + \frac{1}{x}$ as a quadratic equation and solve for x.

8. Find the value of the following infinite continued fraction.

$$3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \ldots}}}}$$

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7-7 Study Guide and Intervention

Operations on Functions

Arithmetic Operations

Operations with Functions	Sum $(f + g)(x) = f(x) + g(x)$ Difference $(f - g)(x) = f(x) - g(x)$ Product $(f \cdot g)(x) = f(x) \cdot g(x)$ Quotient $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Example **Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$.**

$(f + g)(x) = f(x) + g(x)$ Addition of functions
$= (x^2 + 3x - 4) + (3x - 2)$ $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2 + 6x - 6$ Simplify.

$(f - g)(x) = f(x) - g(x)$ Subtraction of functions
$= (x^2 + 3x - 4) - (3x - 2)$ $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2 - 2$ Simplify.

$(f \cdot g)(x) = f(x) \cdot g(x)$ Multiplication of functions
$= (x^2 + 3x - 4)(3x - 2)$ $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2(3x - 2) + 3x(3x - 2) - 4(3x - 2)$ Distributive Property
$= 3x^3 - 2x^2 + 9x^2 - 6x - 12x + 8$ Distributive Property
$= 3x^3 + 7x^2 - 18x + 8$ Simplify.

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ Division of functions
$= \frac{x^2 + 3x - 4}{3x - 2}, x \neq \frac{2}{3}$ $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$

Exercises

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 8x - 3; g(x) = 4x + 5$

2. $f(x) = x^2 + x - 6; g(x) = x - 2$

3. $f(x) = 3x^2 - x + 5; g(x) = 2x - 3$

4. $f(x) = 2x - 1; g(x) = 3x^2 + 11x - 4$

5. $f(x) = x^2 - 1; g(x) = \frac{1}{x + 1}$

Lesson 7-7

7-7 Study Guide and Intervention *(continued)*

Operations on Functions

Composition of Functions

Composition of Functions	Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composite function $f \circ g$ can be described by the equation $[f \circ g](x) = f[g(x)]$.

Example 1 **For $f = \{(1, 2), (3, 3), (2, 4), (4, 1)\}$ and $g = \{(1, 3), (3, 4), (2, 2), (4, 1)\}$, find $f \circ g$ and $g \circ f$ if they exist.**

$f[g(1)] = f(3) = 3$ $\quad f[g(2)] = f(2) = 4$ $\quad f[g(3)] = f(4) = 1$ $\quad f[g(4)] = f(1) = 2$

$f \circ g = \{(1, 3), (2, 4), (3, 1), (4, 2)\}$

$g[f(1)] = g(2) = 2$ $\quad g[f(2)] = g(4) = 1$ $\quad g[f(3)] = g(3) = 4$ $\quad g[f(4)] = g(1) = 3$

$g \circ f = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$

Example 2 **Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = 3x - 4$ and $h(x) = x^2 - 1$.**

$$\begin{aligned}[g \circ h](x) &= g[h(x)] \\ &= g(x^2 - 1) \\ &= 3(x^2 - 1) - 4 \\ &= 3x^2 - 7\end{aligned}$$

$$\begin{aligned}[h \circ g](x) &= h[g(x)] \\ &= h(3x - 4) \\ &= (3x - 4)^2 - 1 \\ &= 9x^2 - 24x + 16 - 1 \\ &= 9x^2 - 24x + 15\end{aligned}$$

Exercises

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

1. $f = \{(-1, 2), (5, 6), (0, 9)\}$,
$g = \{(6, 0), (2, -1), (9, 5)\}$

2. $f = \{(5, -2), (9, 8), (-4, 3), (0, 4)\}$,
$g = \{(3, 7), (-2, 6), (4, -2), (8, 10)\}$

Find $[f \circ g](x)$ and $[g \circ f](x)$.

3. $f(x) = 2x + 7$; $g(x) = -5x - 1$

4. $f(x) = x^2 - 1$; $g(x) = -4x^2$

5. $f(x) = x^2 + 2x$; $g(x) = x - 9$

6. $f(x) = 5x + 4$; $g(x) = 3 - x$

7-7 Skills Practice

Operations on Functions

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = x + 5$

$g(x) = x - 4$

2. $f(x) = 3x + 1$

$g(x) = 2x - 3$

3. $f(x) = x^2$

$g(x) = 4 - x$

4. $f(x) = 3x^2$

$g(x) = \frac{5}{x}$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

5. $f = \{(0, 0), (4, -2)\}$
$g = \{(0, 4), (-2, 0), (5, 0)\}$

6. $f = \{(0, -3), (1, 2), (2, 2)\}$
$g = \{(-3, 1), (2, 0)\}$

7. $f = \{(-4, 3), (-1, 1), (2, 2)\}$
$g = \{(1, -4), (2, -1), (3, -1)\}$

8. $f = \{(6, 6), (-3, -3), (1, 3)\}$
$g = \{(-3, 6), (3, 6), (6, -3)\}$

Find $[g \circ h](x)$ and $[h \circ g](x)$.

9. $g(x) = 2x$
$h(x) = x + 2$

10. $g(x) = -3x$
$h(x) = 4x - 1$

11. $g(x) = x - 6$
$h(x) = x + 6$

12. $g(x) = x - 3$
$h(x) = x^2$

13. $g(x) = 5x$
$h(x) = x^2 + x - 1$

14. $g(x) = x + 2$
$h(x) = 2x^2 - 3$

If $f(x) = 3x$, $g(x) = x + 4$, and $h(x) = x^2 - 1$, find each value.

15. $f[g(1)]$

16. $g[h(0)]$

17. $g[f(-1)]$

18. $h[f(5)]$

19. $g[h(-3)]$

20. $h[f(10)]$

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7-7 Practice

Operations on Functions

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 2x + 1$
$g(x) = x - 3$

2. $f(x) = 8x^2$
$g(x) = \frac{1}{x^2}$

3. $f(x) = x^2 + 7x + 12$
$g(x) = x^2 - 9$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

4. $f = \{(-9, -1), (-1, 0), (3, 4)\}$
$g = \{(0, -9), (-1, 3), (4, -1)\}$

5. $f = \{(-4, 3), (0, -2), (1, -2)\}$
$g = \{(-2, 0), (3, 1)\}$

6. $f = \{(-4, -5), (0, 3), (1, 6)\}$
$g = \{(6, 1), (-5, 0), (3, -4)\}$

7. $f = \{(0, -3), (1, -3), (6, 8)\}$
$g = \{(8, 2), (-3, 0), (-3, 1)\}$

Find $[g \circ h](x)$ and $[h \circ g](x)$.

8. $g(x) = 3x$
$h(x) = x - 4$

9. $g(x) = -8x$
$h(x) = 2x + 3$

10. $g(x) = x + 6$
$h(x) = 3x^2$

11. $g(x) = x + 3$
$h(x) = 2x^2$

12. $g(x) = -2x$
$h(x) = x^2 + 3x + 2$

13. $g(x) = x - 2$
$h(x) = 3x^2 + 1$

If $f(x) = x^2$, $g(x) = 5x$, and $h(x) = x + 4$, find each value.

14. $f[g(1)]$

15. $g[h(-2)]$

16. $h[f(4)]$

17. $f[h(-9)]$

18. $h[g(-3)]$

19. $g[f(8)]$

20. $h[f(20)]$

21. $[f \circ (h \circ g)](-1)$

22. $[f \circ (g \circ h)](4)$

23. BUSINESS The function $f(x) = 1000 - 0.01x^2$ models the manufacturing cost per item when x items are produced, and $g(x) = 150 - 0.001x^2$ models the service cost per item. Write a function $C(x)$ for the total manufacturing and service cost per item.

24. MEASUREMENT The formula $f = \frac{n}{12}$ converts inches n to feet f, and $m = \frac{f}{5280}$ converts feet to miles m. Write a composition of functions that converts inches to miles.

7-7 Reading to Learn Mathematics

Operations on Functions

Pre-Activity Why is it important to combine functions in business?

Read the introduction to Lesson 7-7 at the top of page 383 in your textbook.

Describe two ways to calculate Ms. Coffmon's profit from the sale of 50 birdhouses. (Do not actually calculate her profit.)

Reading the Lesson

1. Determine whether each statement is *true* or *false*. (Remember that *true* means *always* true.)

a. If f and g are polynomial functions, then $f + g$ is a polynomial function.

b. If f and g are polynomial functions, then $\frac{f}{g}$ is a polynomial function.

c. If f and g are polynomial functions, the domain of the function $f \cdot g$ is the set of all real numbers.

d. If $f(x) = 3x + 2$ and $g(x) = x - 4$, the domain of the function $\frac{f}{g}$ is the set of all real numbers.

e. If f and g are polynomial functions, then $(f \circ g)(x) = (g \circ f)(x)$.

f. If f and g are polynomial functions, then $(f \cdot g)(x) = (g \cdot f)(x)$

2. Let $f(x) = 2x - 5$ and $g(x) = x^2 + 1$.

a. Explain in words how you would find $(f \circ g)(-3)$. (Do not actually do any calculations.)

b. Explain in words how you would find $(g \circ f)(-3)$. (Do not actually do any calculations.)

Helping You Remember

3. Some students have trouble remembering the correct order in which to apply the two original functions when evaluating a composite function. Write three sentences, each of which explains how to do this in a slightly different way. (Hint: Use the word *closest* in the first sentence, the words *inside* and *outside* in the second, and the words *left* and *right* in the third.)

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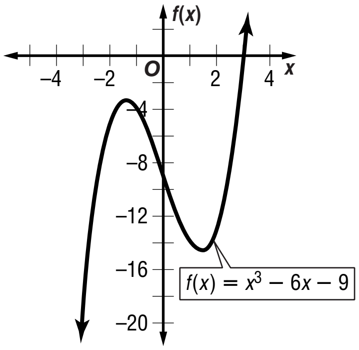
7-7 Enrichment

Relative Maximum Values

The graph of $f(x) = x^3 - 6x - 9$ shows a relative maximum value somewhere between $f(-2)$ and $f(-1)$. You can obtain a closer approximation by comparing values such as those shown in the table.

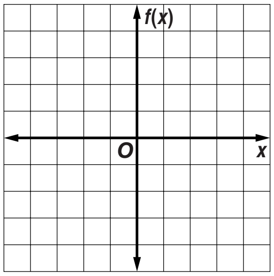
To the nearest tenth a relative maximum value for $f(x)$ is -3.3.



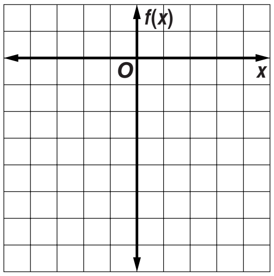
x	f(x)
−2	−5
−1.5	−3.375
−1.4	−3.344
−1.3	−3.397
−1	−4

Using a calculator to find points, graph each function. To the nearest tenth, find a relative maximum value of the function.

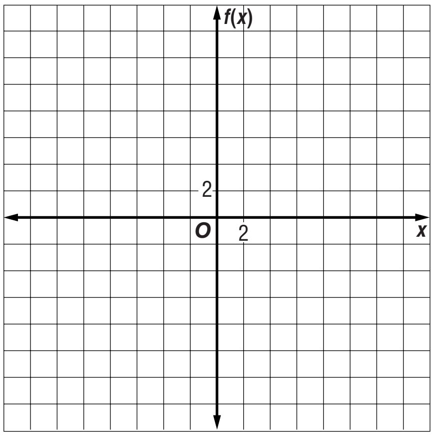
1. $f(x) = x(x^2 - 3)$



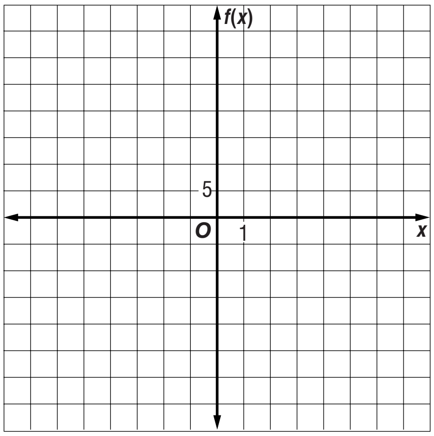
2. $f(x) = x^3 - 3x - 3$



3. $f(x) = x^3 - 9x - 2$



4. $f(x) = x^3 + 2x^2 - 12x - 24$



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7-8 Study Guide and Intervention

Inverse Functions and Relations

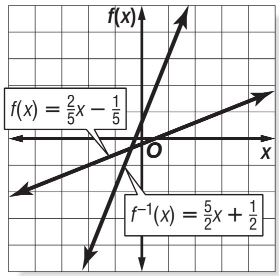
Find Inverses

Inverse Relations	Two relations are inverse relations if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).
Property of Inverse Functions	Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$.

Example **Find the inverse of the function $f(x) = \frac{2}{5}x - \frac{1}{5}$. Then graph the function and its inverse.**

Step 1 Replace $f(x)$ with y in the original equation.

$$f(x) = \frac{2}{5}x - \frac{1}{5} \quad \rightarrow \quad y = \frac{2}{5}x - \frac{1}{5}$$



Step 2 Interchange x and y.

$$x = \frac{2}{5}y - \frac{1}{5}$$

Step 3 Solve for y.

$x = \frac{2}{5}y - \frac{1}{5}$	Inverse
$5x = 2y - 1$	Multiply each side by 5.
$5x + 1 = 2y$	Add 1 to each side.
$\frac{1}{2}(5x + 1) = y$	Divide each side by 2.

The inverse of $f(x) = \frac{2}{5}x - \frac{1}{5}$ is $f^{-1}(x) = \frac{1}{2}(5x + 1)$.

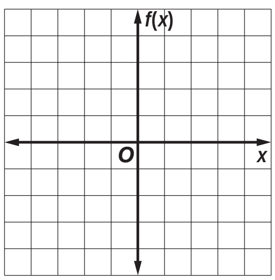
Exercises

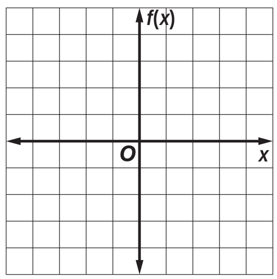
Find the inverse of each function. Then graph the function and its inverse.

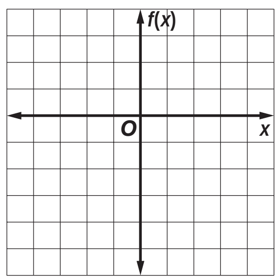
1. $f(x) = \frac{2}{3}x - 1$

2. $f(x) = 2x - 3$

3. $f(x) = \frac{1}{4}x - 2$







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7-8 Study Guide and Intervention *(continued)*

Inverse Functions and Relations

Inverses of Relations and Functions

Inverse Functions	Two functions f and g are inverse functions if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 1 **Determine whether $f(x) = 2x - 7$ and $g(x) = \frac{1}{2}(x + 7)$ are inverse functions.**

$[f \circ g](x) = f[g(x)]$
$= f\left[\frac{1}{2}(x + 7)\right]$
$= 2\left[\frac{1}{2}(x + 7)\right] - 7$
$= x + 7 - 7$
$= x$

$[g \circ f](x) = g[f(x)]$
$= g(2x - 7)$
$= \frac{1}{2}(2x - 7 + 7)$
$= x$

The functions are inverses since both $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 2 **Determine whether $f(x) = 4x + \frac{1}{3}$ and $g(x) = \frac{1}{4}x - 3$ are inverse functions.**

$[f \circ g](x) = f[g(x)]$
$= f\left(\frac{1}{4}x - 3\right)$
$= 4\left(\frac{1}{4}x - 3\right) + \frac{1}{3}$
$= x - 12 + \frac{1}{3}$
$= x - 11\frac{2}{3}$

Since $[f \circ g](x) \neq x$, the functions are not inverses.

Exercises

Determine whether each pair of functions are inverse functions.

1. $f(x) = 3x - 1$
$g(x) = \frac{1}{3}x + \frac{1}{3}$

2. $f(x) = \frac{1}{4}x + 5$
$g(x) = 4x - 20$

3. $f(x) = \frac{1}{2}x - 10$
$g(x) = 2x + \frac{1}{10}$

4. $f(x) = 2x + 5$
$g(x) = 5x + 2$

5. $f(x) = 8x - 12$
$g(x) = \frac{1}{8}x + 12$

6. $f(x) = -2x + 3$
$g(x) = -\frac{1}{2}x + \frac{3}{2}$

7. $f(x) = 4x - \frac{1}{2}$
$g(x) = \frac{1}{4}x + \frac{1}{8}$

8. $f(x) = 2x - \frac{3}{5}$
$g(x) = \frac{1}{10}(5x + 3)$

9. $f(x) = 4x + \frac{1}{2}$
$g(x) = \frac{1}{2}x - \frac{3}{2}$

10. $f(x) = 10 - \frac{x}{2}$
$g(x) = 20 - 2x$

11. $f(x) = 4x - \frac{4}{5}$
$g(x) = \frac{x}{4} + \frac{1}{5}$

12. $f(x) = 9 + \frac{3}{2}x$
$g(x) = \frac{2}{3}x - 6$

NAME ______________________________ DATE __________ PERIOD _____

7-8 Skills Practice

Inverse Functions and Relations

Find the inverse of each relation.

1. $\{(3, 1), (4, -3), (8, -3)\}$

2. $\{(-7, 1), (0, 5), (5, -1)\}$

3. $\{(-10, -2), (-7, 6), (-4, -2), (-4, 0)\}$

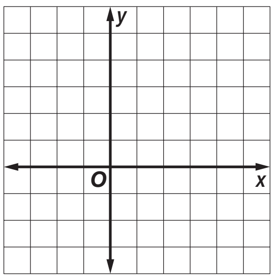
4. $\{(0, -9), (5, -3), (6, 6), (8, -3)\}$

5. $\{(-4, 12), (0, 7), (9, -1), (10, -5)\}$

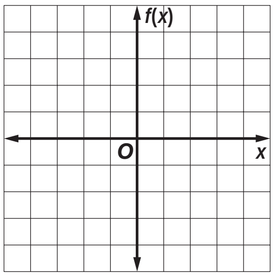
6. $\{(-4, 1), (-4, 3), (0, -8), (8, -9)\}$

Find the inverse of each function. Then graph the function and its inverse.

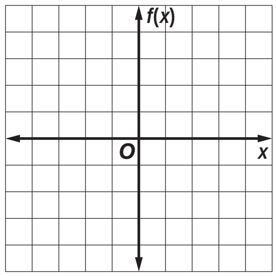
7. $y = 4$



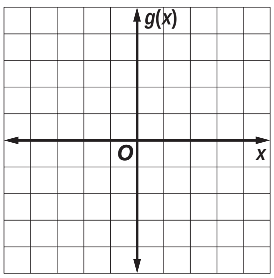
8. $f(x) = 3x$



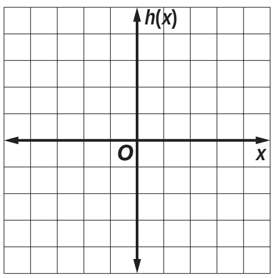
9. $f(x) = x + 2$



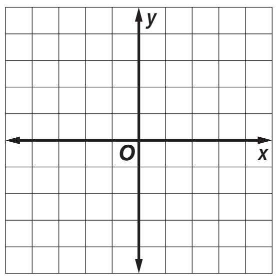
10. $g(x) = 2x - 1$



11. $h(x) = \frac{1}{4}x$



12. $y = \frac{2}{3}x + 2$



Determine whether each pair of functions are inverse functions.

13. $f(x) = x - 1$
$g(x) = 1 - x$

14. $f(x) = 2x + 3$
$g(x) = \frac{1}{2}(x - 3)$

15. $f(x) = 5x - 5$
$g(x) = \frac{1}{5}x + 1$

16. $f(x) = 2x$
$g(x) = \frac{1}{2}x$

17. $h(x) = 6x - 2$
$g(x) = \frac{1}{6}x + 3$

18. $f(x) = 8x - 10$
$g(x) = \frac{1}{8}x + \frac{5}{4}$

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7-8 Practice

Inverse Functions and Relations

Find the inverse of each relation.

1. $\{(0, 3), (4, 2), (5, -6)\}$

2. $\{(-5, 1), (-5, -1), (-5, 8)\}$

3. $\{(-3, -7), (0, -1), (5, 9), (7, 13)\}$

4. $\{(8, -2), (10, 5), (12, 6), (14, 7)\}$

5. $\{(-5, -4), (1, 2), (3, 4), (7, 8)\}$

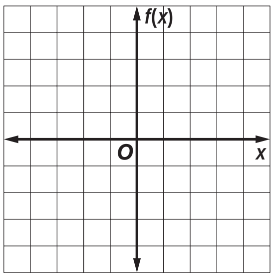
6. $\{(-3, 9), (-2, 4), (0, 0), (1, 1)\}$

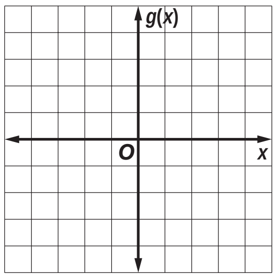
Find the inverse of each function. Then graph the function and its inverse.

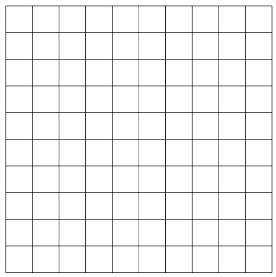
7. $f(x) = \frac{3}{4}x$

8. $g(x) = 3 + x$

9. $y = 3x - 2$







Determine whether each pair of functions are inverse functions.

10. $f(x) = x + 6$
$g(x) = x - 6$

11. $f(x) = -4x + 1$
$g(x) = \frac{1}{4}(1 - x)$

12. $g(x) = 13x - 13$
$h(x) = \frac{1}{13}x - 1$

13. $f(x) = 2x$
$g(x) = -2x$

14. $f(x) = \frac{6}{7}x$
$g(x) = \frac{7}{6}x$

15. $g(x) = 2x - 8$
$h(x) = \frac{1}{2}x + 4$

16. MEASUREMENT The points (63, 121), (71, 180), (67, 140), (65, 108), and (72, 165) give the weight in pounds as a function of height in inches for 5 students in a class. Give the points for these students that represent height as a function of weight.

REMODELING For Exercises 17 and 18, use the following information.

The Clearys are replacing the flooring in their 15 foot by 18 foot kitchen. The new flooring costs $17.99 per square yard. The formula $f(x) = 9x$ converts square yards to square feet.

17. Find the inverse $f^{-1}(x)$. What is the significance of $f^{-1}(x)$ for the Clearys?

18. What will the new flooring cost the Cleary's?

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7-8 Reading to Learn Mathematics

Inverse Functions and Relations

Pre-Activity **How are inverse functions related to measurement conversions?**

Read the introduction to Lesson 7-8 at the top of page 390 in your textbook.

A function multiplies a number by 3 and then adds 5 to the result. What does the inverse function do, and in what order?

Reading the Lesson

1. Complete each statement.

a. If two relations are inverses, the domain of one relation is the ________________ of the other.

b. Suppose that $g(x)$ is a relation and that the point $(4, -2)$ is on its graph. Then a point on the graph of $g^{-1}(x)$ is ________________.

c. The ________________ test can be used on the graph of a function to determine whether the function has an inverse function.

d. If you are given the graph of a function, you can find the graph of its inverse by reflecting the original graph over the line with equation ________________.

e. If f and g are inverse functions, then $(f \circ g)(x) =$ ________________ and $(g \circ f)(x) =$ ________________.

f. A function has an inverse that is also a function only if the given function is ________________.

g. Suppose that $h(x)$ is a function whose inverse is also a function. If $h(5) = 12$, then $h^{-1}(12) =$ ________________.

2. Assume that $f(x)$ is a one-to-one function defined by an algebraic equation. Write the four steps you would follow in order to find the equation for $f^{-1}(x)$.

1. ______________________________
2. ______________________________
3. ______________________________
4. ______________________________

Helping You Remember

3. A good way to remember something new is to relate it to something you already know. How are the vertical and horizontal line tests related?

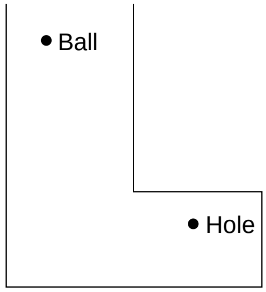
Lesson 7-8

NAME ____ DATE ____________ PERIOD _____

7-8 Enrichment

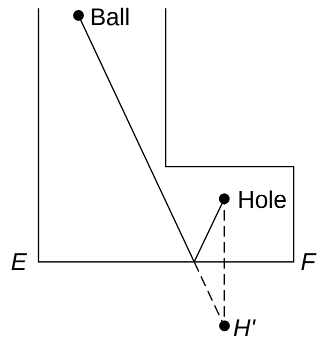
Miniature Golf

In miniature golf, the object of the game is to roll the golf ball into the hole in as few shots as possible. As in the diagram at the right, the hole is often placed so that a direct shot is impossible. Reflections can be used to help determine the direction that the ball should be rolled in order to score a hole-in-one.



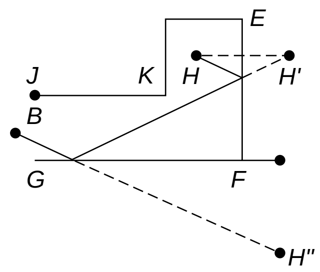
Example 1 **Using wall $\overline{EF}$, find the path to use to score a hole-in-one.**

Find the reflection image of the "hole" with respect to $\overline{EF}$ and label it H'. The intersection of $\overline{BH'}$ with wall $\overline{EF}$ is the point at which the shot should be directed.



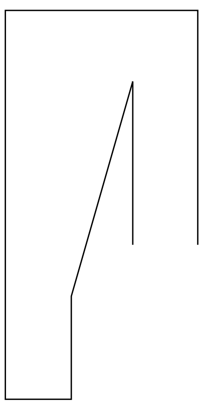
Example 2 **For the hole at the right, find a path to score a hole-in-one.**

Find the reflection image of H with respect to $\overline{EF}$ and label it H'. In this case, $\overline{BH'}$ intersects $\overline{JK}$ before intersecting $\overline{EF}$. Thus, this path cannot be used. To find a usable path, find the reflection image of H' with respect to $\overline{GF}$ and label it H''. Now, the intersection of $\overline{BH''}$ with wall $\overline{GF}$ is the point at which the shot should be directed.

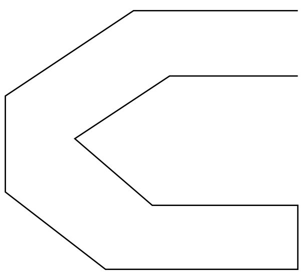


Copy each figure. Then, use reflections to determine a possible path for a hole-in-one.

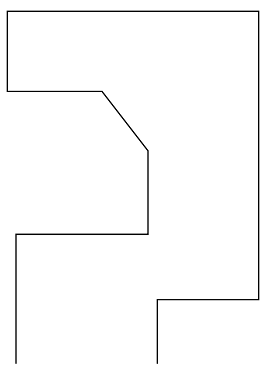
1.



2.



3.



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7-9 Study Guide and Intervention

Square Root Functions and Inequalities

Square Root Functions A function that contains the square root of a variable expression is a **square root function**.

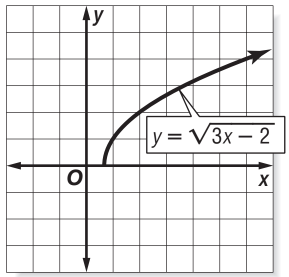
Example **Graph $y = \sqrt{3x - 2}$. State its domain and range.**

Since the radicand cannot be negative, $3x - 2 \geq 0$ or $x \geq \frac{2}{3}$.

The x-intercept is $\frac{2}{3}$. The range is $y \geq 0$.

Make a table of values and graph the function.

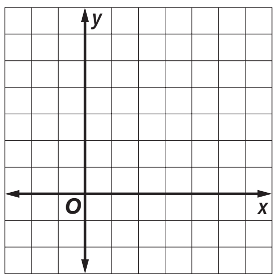
x	y
$\frac{2}{3}$	0
1	1
2	2
3	$\sqrt{7}$



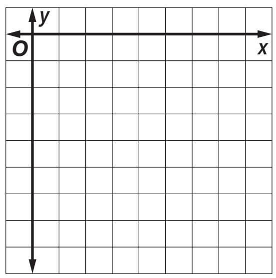
Exercises

Graph each function. State the domain and range of the function.

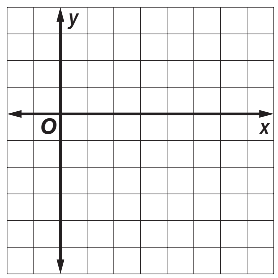
1. $y = \sqrt{2x}$



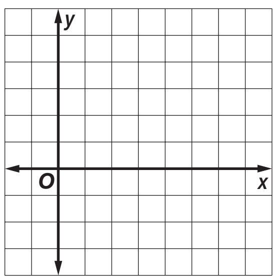
2. $y = -3\sqrt{x}$



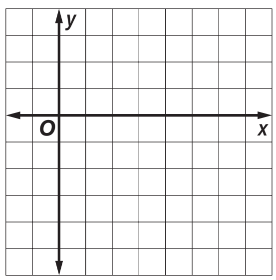
3. $y = -\sqrt{\frac{x}{2}}$



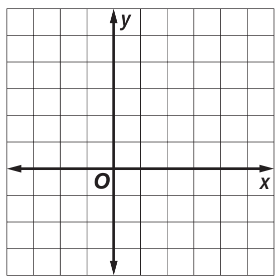
4. $y = 2\sqrt{x - 3}$



5. $y = -\sqrt{2x - 3}$



6. $y = \sqrt{2x + 5}$



Lesson 7-9

7-9 Study Guide and Intervention *(continued)*

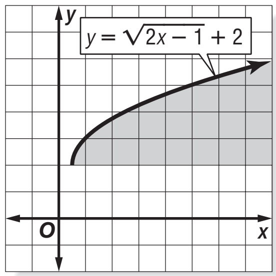
Square Root Functions and Inequalities

Square Root Inequalities A **square root inequality** is an inequality that contains the square root of a variable expression. Use what you know about graphing square root functions and quadratic inequalities to graph square root inequalities.

Example **Graph $y \leq \sqrt{2x - 1} + 2$.**

Graph the related equation $y = \sqrt{2x - 1} + 2$. Since the boundary should be included, the graph should be solid.

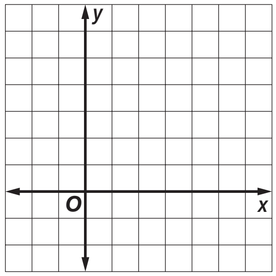
The domain includes values for $x \geq \frac{1}{2}$, so the graph is to the right of $x = \frac{1}{2}$. The range includes only numbers greater than 2, so the graph is above $y = 2$.



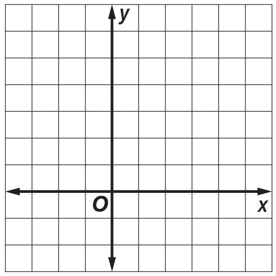
Exercises

Graph each inequality.

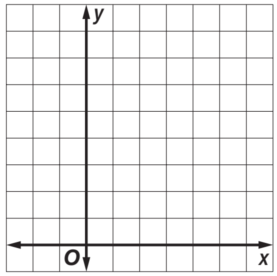
1. $y < 2\sqrt{x}$



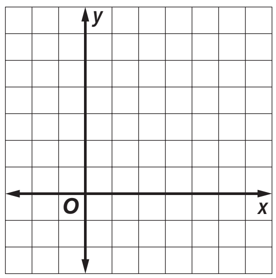
2. $y > \sqrt{x + 3}$



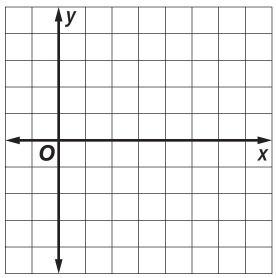
3. $y < 3\sqrt{2x - 1}$



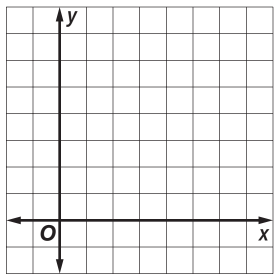
4. $y < \sqrt{3x - 4}$



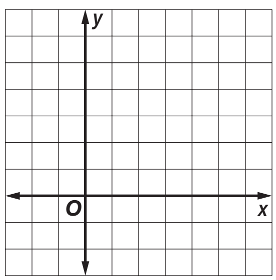
5. $y \geq \sqrt{x + 1} - 4$



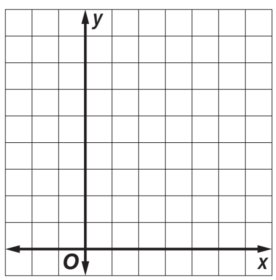
6. $y > 2\sqrt{2x - 3}$



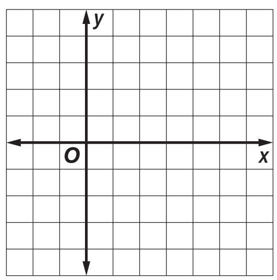
7. $y \geq \sqrt{3x + 1} - 2$



8. $y \leq \sqrt{4x - 2} + 1$



9. $y < 2\sqrt{2x - 1} - 4$



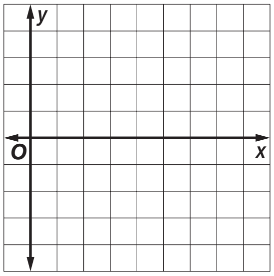
NAME ______________________________ DATE ____________ PERIOD _____

7-9 Skills Practice

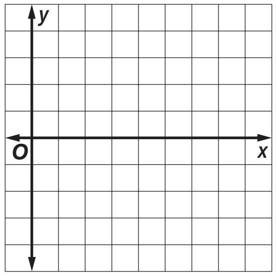
Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

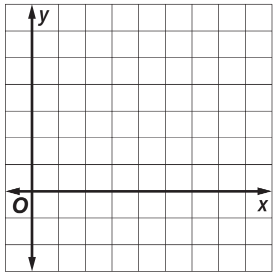
1. $y = \sqrt{2x}$



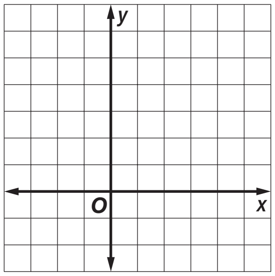
2. $y = -\sqrt{3x}$



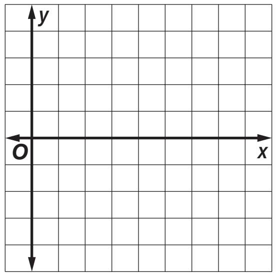
3. $y = 2\sqrt{x}$



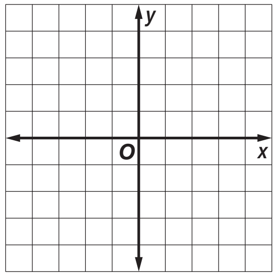
4. $y = \sqrt{x + 3}$



5. $y = -\sqrt{2x - 5}$

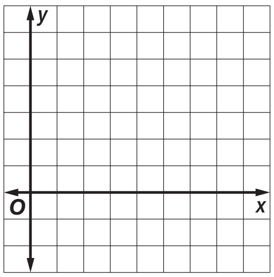


6. $y = \sqrt{x + 4} - 2$

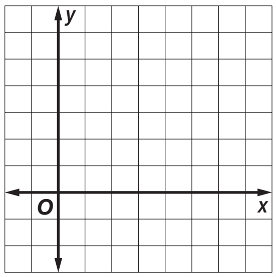


Graph each inequality.

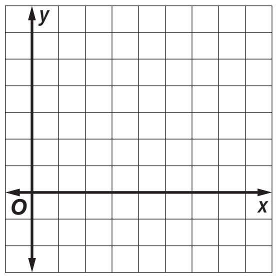
7. $y < \sqrt{4x}$



8. $y \geq \sqrt{x + 1}$



9. $y \leq \sqrt{4x - 3}$



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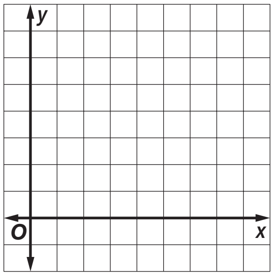
7-9 Practice

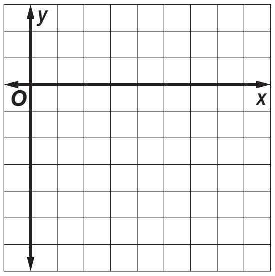
Square Root Functions and Inequalities

Graph each function. State the domain and range of each function.

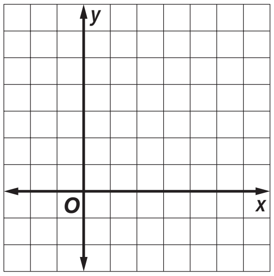
1. $y = \sqrt{5x}$



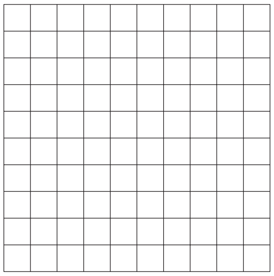
2. $y = -\sqrt{x - 1}$



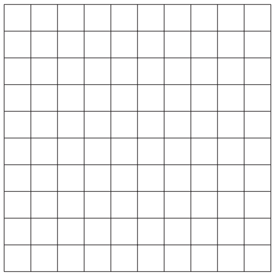
3. $y = 2\sqrt{x + 2}$



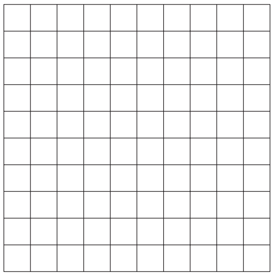
4. $y = \sqrt{3x - 4}$



5. $y = \sqrt{x + 7} - 4$

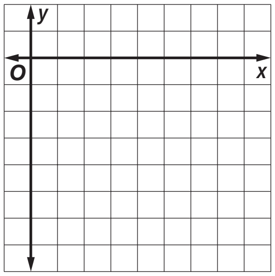


6. $y = 1 - \sqrt{2x + 3}$

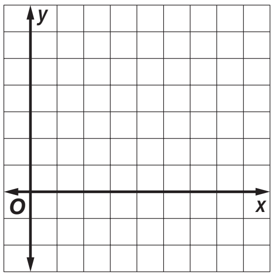


Graph each inequality.

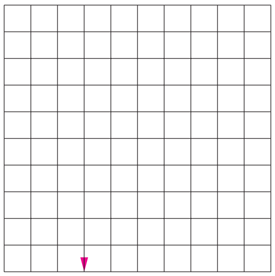
7. $y \geq -\sqrt{6x}$



8. $y \leq \sqrt{x - 5} + 3$



9. $y > -2\sqrt{3x + 2}$



10. ROLLER COASTERS The velocity of a roller coaster as it moves down a hill is $v = \sqrt{v_0^2 + 64h}$, where v_0 is the initial velocity and h is the vertical drop in feet. If $v = 70$ feet per second and $v_0 = 8$ feet per second, find h.

11. WEIGHT Use the formula $d = \sqrt{\frac{3960^2 W_E}{W_s}} - 3960$, which relates distance from Earth d in miles to weight. If an astronaut's weight on Earth W_E is 148 pounds and in space W_s is 115 pounds, how far from Earth is the astronaut?

7-9 Reading to Learn Mathematics

Square Root Functions

Pre-Activity **How are square root functions used in bridge design?**

Read the introduction to Lesson 7-9 at the top of page 395 in your textbook.

If the weight to be supported by a steel cable is doubled, should the diameter of the support cable also be doubled? If not, by what number should the diameter be multiplied?

Reading the Lesson

1. Match each square root function from the list on the left with its domain and range from the list on the right.

a. $y = \sqrt{x}$ **i.** domain: $x \geq 0$; range: $y \geq 3$

b. $y = \sqrt{x + 3}$ **ii.** domain: $x \geq 0$; range: $y \leq 0$

c. $y = \sqrt{x} + 3$ **iii.** domain: $x \geq 0$; range: $y \leq -3$

d. $y = \sqrt{x - 3}$ **iv.** domain: $x \geq 0$; range: $y \geq 0$

e. $y = -\sqrt{x}$ **v.** domain: $x \geq 3$; range: $y \geq 0$

f. $y = -\sqrt{x - 3}$ **vi.** domain: $x \leq 3$; range: $y \geq 3$

g. $y = \sqrt{3 - x} + 3$ **vii.** domain: $x \geq 3$; range: $y \leq 0$

h. $y = -\sqrt{x} - 3$ **viii.** domain: $x \geq -3$; range: $y \geq 0$

2. The graph of the inequality $y \leq \sqrt{3x + 6}$ is a shaded region. Which of the following points lie inside this region?

(3, 0) (2, 4) (5, 2) (4, −2) (6, 6)

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose you are studying this lesson with a classmate who thinks that you cannot have square root functions because every positive real number has two square roots. How would you explain the idea of square root functions to your classmate?

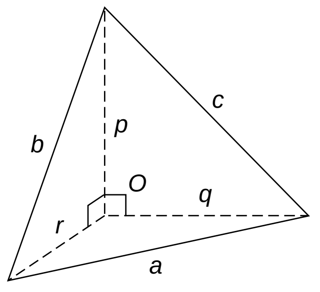
NAME ______________________________ DATE ____________ PERIOD _____

7-9 Enrichment

Reading Algebra

If two mathematical problems have basic structural similarities, they are said to be **analogous.** Using analogies is one way of discovering and proving new theorems.

The following numbered sentences discuss a three-dimensional analogy to the Pythagorean theorem.



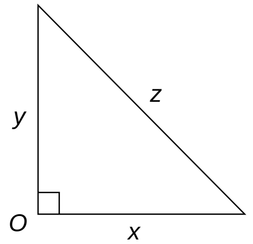
01 Consider a tetrahedron with three perpendicular faces that meet at vertex *O*.

02 Suppose you want to know how the areas *A, B,* and *C* of the three faces that meet at vertex *O* are related to the area *D* of the face opposite vertex *O*.

03 It is natural to expect a formula analogous to the Pythagorean theorem $z^2 = x^2 + y^2$, which is true for a similar situation in two dimensions.

04 To explore the three-dimensional case, you might guess a formula and then try to prove it.

05 Two reasonable guesses are $D^3 = A^3 + B^3 + C^3$ and $D^2 = A^2 + B^2 + C^2$.



Refer to the numbered sentences to answer the questions.

1. Use sentence 01 and the top diagram. The prefix *tetra-* means four. Write an informal definition of tetrahedron.

2. Use sentence 02 and the top diagram. What are the lengths of the sides of each face of the tetrahedron?

3. Rewrite sentence 01 to state a two-dimensional analogue.

4. Refer to the top diagram and write expressions for the areas *A, B,* and *C*

5. To explore the three-dimensional case, you might begin by expressing *a, b,* and *c* in terms of *p, q,* and *r.* Use the Pythagorean theorem to do this.

6. Which guess in sentence 05 seems more likely? Justify your answer.