GLENCOE MATHEMATICS

Algebra 2

Chapter 4
Resource Masters

Glencoe McGraw-Hill

New York, New York
Columbus, Ohio
Chicago, Illinois
Peoria, Illinois
Woodland Hills, California

Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-828029-X
Skills Practice Workbook	0-07-828023-0
Practice Workbook	0-07-828024-9

ANSWERS FOR WORKBOOKS The answers for Chapter 4 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

Glencoe/McGraw-Hill
A Division of The McGraw-Hill Companies

Send all inquiries to:
The McGraw-Hill Companies
8787 Orion Place
Columbus, OH 43240-4027

ISBN: 0-07-828007-9

Algebra 2
Chapter 4 Resource Masters

1 2 3 4 5 6 7 8 9 10 066 11 10 09 08 07 06 05 04 03 02

Contents

Teacher's Guide to Using the Chapter 4 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 4 Resource Masters* includes the core materials needed for Chapter 4. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 2 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 4-1. Encourage them to add these pages to their Algebra 2 Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention
Each lesson in *Algebra 2* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics
One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 4 Resource Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 2. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 216–217. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

4 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 4. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
Cramer's Rule KRAY·muhrs		
determinant		
dilation dy·LAY·shuhn		
element		
expansion by minors		
identity matrix		
image		
inverse		
isometry eye·SAH·muh·tree		
matrix MAY·trihks		

(continued on the next page)

4

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
matrix equation		
preimage		
reflection		
rotation		
scalar multiplication SKAY·luhr		
square matrix		
transformation		
translation		
vertex matrix		
zero matrix		

NAME ____________________ DATE ________ PERIOD _____

4-1 Study Guide and Intervention

Introduction to Matrices

Organize Data

Matrix	a rectangular array of variables or constants in horizontal rows and vertical columns, usually enclosed in brackets.

A matrix can be described by its **dimensions**. A matrix with m rows and n columns is an $m \times n$ matrix.

Example 1 **Owls' eggs incubate for 30 days and their fledgling period is also 30 days. Swifts' eggs incubate for 20 days and their fledgling period is 44 days. Pigeon eggs incubate for 15 days, and their fledgling period is 17 days. Eggs of the king penguin incubate for 53 days, and the fledgling time for a king penguin is 360 days. Write a 2 × 4 matrix to organize this information.** **Source:** *The Cambridge Factfinder*

$$\begin{array}{r} \\ \text{Incubation} \\ \text{Fledgling} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Owl} & \text{Swift} & \text{Pigeon} & \text{King Penguin} \end{array} \\ \begin{bmatrix} 30 & 20 & 15 & 53 \\ 30 & 44 & 17 & 360 \end{bmatrix} \end{array}$$

Example 2 **What are the dimensions of matrix A if $A = \begin{bmatrix} 13 & 10 & -3 & 45 \\ 2 & 8 & 15 & 80 \end{bmatrix}$?**

Since matrix A has 2 rows and 4 columns, the dimensions of A are 2×4.

Exercises

State the dimensions of each matrix.

1. $\begin{bmatrix} 15 & 5 & 27 & -4 \\ 23 & 6 & 0 & 5 \\ 14 & 70 & 24 & -3 \\ 63 & 3 & 42 & 90 \end{bmatrix}$

2. $[16 \quad 12 \quad 0]$

3. $\begin{bmatrix} 71 & 44 \\ 39 & 27 \\ 45 & 16 \\ 92 & 53 \\ 78 & 65 \end{bmatrix}$

4. A travel agent provides for potential travelers the normal high temperatures for the months of January, April, July, and October for various cities. In Boston these figures are 36°, 56°, 82°, and 63°. In Dallas they are 54°, 76°, 97°, and 79°. In Los Angeles they are 68°, 72°, 84°, and 79°. In Seattle they are 46°, 58°, 74°, and 60°, and in St. Louis they are 38°, 67°, 89°, and 69°. Organize this information in a 4 × 5 matrix. **Source:** *The New York Times Almanac*

Lesson 4-1

4-1 Study Guide and Intervention *(continued)*

Introduction to Matrices

Equations Involving Matrices

Equal Matrices	Two matrices are equal if they have the same dimensions and each element of one matrix is equal to the corresponding element of the other matrix.

You can use the definition of equal matrices to solve matrix equations.

Example **Solve** $\begin{bmatrix} 4x \\ y \end{bmatrix} = \begin{bmatrix} -2y + 2 \\ x - 8 \end{bmatrix}$ **for x and y.**

Since the matrices are equal, the corresponding elements are equal. When you write the sentences to show the equality, two linear equations are formed.

$4x = -2y + 2$
$y = x - 8$

This system can be solved using substitution.

$4x = -2y + 2$	First equation
$4x = -2(x - 8) + 2$	Substitute $x - 8$ for y.
$4x = -2x + 16 + 2$	Distributive Property
$6x = 18$	Add $2x$ to each side.
$x = 3$	Divide each side by 6.

To find the value of y, substitute 3 for x in either equation.

$y = x - 8$	Second equation
$y = 3 - 8$	Substitute 3 for x.
$y = -5$	Subtract.

The solution is $(3, -5)$.

Exercises

Solve each equation.

1. $[5x \; 4y] = [20 \; 20]$

2. $\begin{bmatrix} 3x \\ y \end{bmatrix} = \begin{bmatrix} 28 + 4y \\ -3x - 2 \end{bmatrix}$

3. $\begin{bmatrix} -2y \\ x \end{bmatrix} = \begin{bmatrix} 4 - 5x \\ y + 5 \end{bmatrix}$

4. $\begin{bmatrix} x - 2y \\ 3x - 4y \end{bmatrix} = \begin{bmatrix} -1 \\ 22 \end{bmatrix}$

5. $\begin{bmatrix} 2x + 3y \\ x - 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 12 \end{bmatrix}$

6. $\begin{bmatrix} 5x + 3y \\ 2x - y \end{bmatrix} = \begin{bmatrix} -1 \\ -18 \end{bmatrix}$

7. $\begin{bmatrix} 8x - y & 16x \\ 12 & y - 4x \end{bmatrix} = \begin{bmatrix} 18 & 20 \\ 12 & -13 \end{bmatrix}$

8. $\begin{bmatrix} 8x - 6y \\ 12x + 4y \end{bmatrix} = \begin{bmatrix} -3 \\ -11 \end{bmatrix}$

9. $\begin{bmatrix} \frac{x}{3} + \frac{y}{7} \\ \frac{x}{2} + 2y \end{bmatrix} = \begin{bmatrix} 9 \\ 51 \end{bmatrix}$

10. $\begin{bmatrix} 3x + 1.5 \\ 2y - 2.4 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 8.0 \end{bmatrix}$

11. $\begin{bmatrix} 2x + 3y \\ -4x + 0.5y \end{bmatrix} = \begin{bmatrix} 17 \\ -8 \end{bmatrix}$

12. $\begin{bmatrix} x - y \\ x + y \end{bmatrix} = \begin{bmatrix} 0 \\ -25 \end{bmatrix}$

NAME ______________________________ DATE ____________ PERIOD _____

4-1 Skills Practice

Introduction to Matrices

Lesson 4-1

State the dimensions of each matrix.

1. $\begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix}$

2. $[0 \quad 15]$

3. $\begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}$

4. $\begin{bmatrix} 6 & 1 & 2 \\ -3 & 4 & 5 \\ -2 & 7 & 9 \end{bmatrix}$

5. $\begin{bmatrix} 9 & 3 & -3 & -6 \\ 3 & 4 & -4 & 5 \end{bmatrix}$

6. $\begin{bmatrix} -1 \\ -1 \\ -1 \\ -3 \end{bmatrix}$

Solve each equation.

7. $[5x \quad 3y] = [15 \quad 12]$

8. $[3x - 2] = [7]$

9. $\begin{bmatrix} 7x \\ 14 \end{bmatrix} = \begin{bmatrix} -14 \\ 2y \end{bmatrix}$

10. $[2x \quad -8y \quad z] = [10 \quad 16 \quad -1]$

11. $\begin{bmatrix} 8 - x \\ 2y - 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

12. $\begin{bmatrix} 20 \\ 56 - 6y \end{bmatrix} = \begin{bmatrix} 10x \\ 32 \end{bmatrix}$

13. $\begin{bmatrix} 5x \\ 24 \end{bmatrix} = \begin{bmatrix} -20 \\ 8y \end{bmatrix}$

14. $\begin{bmatrix} 3x + 2 \\ 7y - 2 \end{bmatrix} = \begin{bmatrix} 5x + 2 \\ 3y - 10 \end{bmatrix}$

15. $\begin{bmatrix} 4x - 1 \\ 9y + 5 \end{bmatrix} = \begin{bmatrix} 3x \\ y - 3 \end{bmatrix}$

16. $\begin{bmatrix} 3x + 1 & 18 \\ 12 & 4z \end{bmatrix} = \begin{bmatrix} 7 & 2y - 4 \\ 12 & 28 \end{bmatrix}$

17. $\begin{bmatrix} x \\ 16 \\ 3z \end{bmatrix} = \begin{bmatrix} 9 \\ 4y \\ 9 \end{bmatrix}$

18. $\begin{bmatrix} 5x \\ 4y - 3 \\ 8z \end{bmatrix} = \begin{bmatrix} 4x + 1 \\ 13 \\ 4z \end{bmatrix}$

19. $\begin{bmatrix} 2x \\ y + 2 \end{bmatrix} = \begin{bmatrix} 6y \\ x \end{bmatrix}$

20. $\begin{bmatrix} x \\ 3y \end{bmatrix} = \begin{bmatrix} 4y \\ x - 3 \end{bmatrix}$

NAME ______________________________ DATE ____________ PERIOD _____

4-1 Practice

Introduction to Matrices

State the dimensions of each matrix.

1. $\begin{bmatrix} -3 & -3 & 7 \end{bmatrix}$

2. $\begin{bmatrix} 5 & 8 & -1 \\ -2 & 1 & 8 \end{bmatrix}$

3. $\begin{bmatrix} -2 & 2 & -2 & 3 \\ 5 & 16 & 0 & 0 \\ 4 & 7 & -1 & 4 \end{bmatrix}$

Solve each equation.

4. $\begin{bmatrix} 4x & 42 \end{bmatrix} = \begin{bmatrix} 24 & 6y \end{bmatrix}$

5. $\begin{bmatrix} -2x & 22 & -3z \end{bmatrix} = \begin{bmatrix} 6x & -2y & 45 \end{bmatrix}$

6. $\begin{bmatrix} 6x \\ 2y + 3 \end{bmatrix} = \begin{bmatrix} -36 \\ 17 \end{bmatrix}$

7. $\begin{bmatrix} 7x - 8 \\ 8y - 3 \end{bmatrix} = \begin{bmatrix} 20 \\ 2y + 3 \end{bmatrix}$

8. $\begin{bmatrix} -4x - 3 \\ 6y \end{bmatrix} = \begin{bmatrix} -3x \\ -2y + 16 \end{bmatrix}$

9. $\begin{bmatrix} 6x - 12 \\ -3y + 6 \end{bmatrix} = \begin{bmatrix} -3x - 21 \\ 8y - 5 \end{bmatrix}$

10. $\begin{bmatrix} -5 & 3x + 1 \\ 2y - 1 & 3z - 2 \end{bmatrix} = \begin{bmatrix} -5 & x - 1 \\ 3y & 5z - 4 \end{bmatrix}$

11. $\begin{bmatrix} 3x \\ y + 4 \end{bmatrix} = \begin{bmatrix} y + 8 \\ 17 \end{bmatrix}$

12. $\begin{bmatrix} 5x + 8y \\ 3x - 11 \end{bmatrix} = \begin{bmatrix} -1 \\ y \end{bmatrix}$

13. $\begin{bmatrix} 2x + y \\ 3x + 2y \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

14. TICKET PRICES The table at the right gives ticket prices for a concert. Write a 2×3 matrix that represents the cost of a ticket.

	Child	Student	Adult
Cost Purchased in Advance	$6	$12	$18
Cost Purchased at the Door	$8	$15	$22

CONSTRUCTION For Exercises 15 and 16, use the following information.

During each of the last three weeks, a road-building crew has used three truck-loads of gravel. The table at the right shows the amount of gravel in each load.

Week 1	Week 2	Week 3
Load 1 40 tons	Load 1 40 tons	Load 1 32 tons
Load 2 32 tons	Load 2 40 tons	Load 2 24 tons
Load 3 24 tons	Load 3 32 tons	Load 3 24 tons

15. Write a matrix for the amount of gravel in each load.

16. What are the dimensions of the matrix?

NAME ____ DATE ____________ PERIOD _____

4-1 Reading to Learn Mathematics

Introduction to Matrices

Pre-Activity **How are matrices used to make decisions?**

Read the introduction to Lesson 4-1 at the top of page 154 in your textbook.

What is the base price of a Mid-Size SUV?

Lesson 4-1

Reading the Lesson

1. Give the dimensions of each matrix.

a. $\begin{bmatrix} 3 & 2 & 5 \\ -1 & 0 & 6 \end{bmatrix}$ **b.** $[1 \quad 4 \quad 0 \quad -8 \quad 2]$

2. Identify each matrix with as many of the following descriptions that apply: *row matrix*, *column matrix*, *square matrix*, *zero matrix.*

a. $[6 \quad 5 \quad 4 \quad 3]$

b. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

c. $[0]$

3. Write a system of equations that you could use to solve the following matrix equation for x, y, and z. (Do not actually solve the system.)

$$\begin{bmatrix} 3x \\ x + y \\ y - z \end{bmatrix} = \begin{bmatrix} -9 \\ 5 \\ 6 \end{bmatrix}$$

Helping You Remember

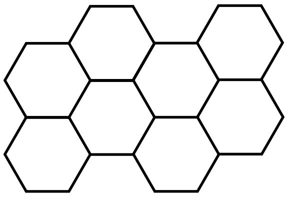
4. Some students have trouble remembering which number comes first in writing the dimensions of a matrix. Think of an easy way to remember this.

NAME ______________________________ DATE ____________ PERIOD _____

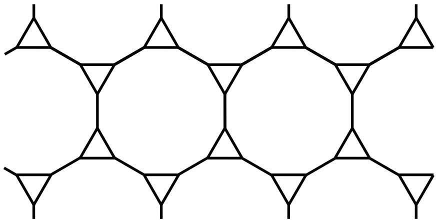
4-1 Enrichment

Tessellations

A **tessellation** is an arangement of polygons covering a plane without any gaps or overlapping. One example of a tessellation is a honeycomb. Three congruent regular hexagons meet at each vertex, and there is no wasted space between cells. This tessellation is called a regular tessellation since it is formed by congruent regular polygons.



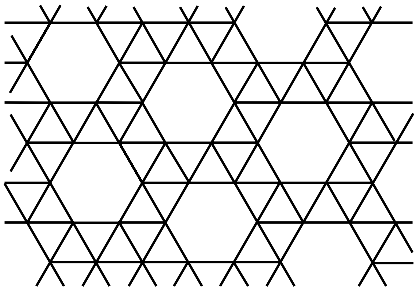
A **semi-regular tessellation** is a tessellation formed by two or more regular polygons such that the number of sides of the polygons meeting at each vertex is the same.



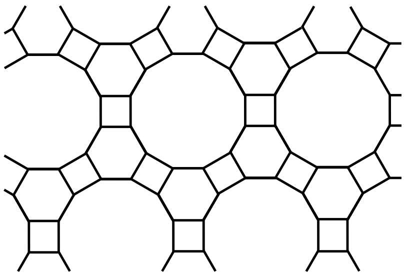
For example, the tessellation at the left has two regular dodecagons and one equilateral triangle meeting at each vertex. We can name this tessellation a 3-12-12 for the number of sides of each polygon that meet at one vertex.

Name each semi-regular tessellation shown according to the number of sides of the polygons that meet at each vertex.

1.

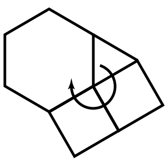


2.

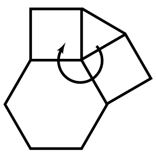


An equilateral triangle, two squares, and a regular hexagon can be used to surround a point in two different orders. Continue each pattern to see which is a semi-regular tessellation.

3. 3-4-4-6



4. 3-4-6-4



On another sheet of paper, draw part of each design. Then determine if it is a semi-regular tessellation.

5. 3-3-4-12 **6.** 3-4-3-12 **7.** 4-8-8 **8.** 3-3-3-4-4

4-2 Study Guide and Intervention

Operations with Matrices

Add and Subtract Matrices

Addition of Matrices	$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} + \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} a+j & b+k & c+l \\ d+m & e+n & f+o \\ g+p & h+q & i+r \end{bmatrix}$
Subtraction of Matrices	$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} - \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} a-j & b-k & c-l \\ d-m & e-n & f-o \\ g-p & h-q & i-r \end{bmatrix}$

Example 1 **Find $A + B$ if $A = \begin{bmatrix} 6 & -7 \\ 2 & -12 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ -5 & -6 \end{bmatrix}$.**

$$A + B = \begin{bmatrix} 6 & -7 \\ 2 & -12 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ -5 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 6+4 & -7+2 \\ 2+(-5) & -12+(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 \\ -3 & -18 \end{bmatrix}$$

Example 2 **Find $A - B$ if $A = \begin{bmatrix} -2 & 8 \\ 3 & -4 \\ 10 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ -2 & 1 \\ -6 & 8 \end{bmatrix}$.**

$$A - B = \begin{bmatrix} -2 & 8 \\ 3 & -4 \\ 10 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -3 \\ -2 & 1 \\ -6 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -2-4 & 8-(-3) \\ 3-(-2) & -4-1 \\ 10-(-6) & 7-8 \end{bmatrix} = \begin{bmatrix} -6 & 11 \\ 5 & -5 \\ 16 & -1 \end{bmatrix}$$

Exercises

Perform the indicated operations. If the matrix does not exist, write *impossible*.

1. $\begin{bmatrix} 8 & 7 \\ -10 & -6 \end{bmatrix} - \begin{bmatrix} -4 & 3 \\ 2 & -12 \end{bmatrix}$

2. $\begin{bmatrix} 6 & -5 & 9 \\ -3 & 4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 3 & 2 \\ 6 & 9 & -4 \end{bmatrix}$

3. $\begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 & -2 \end{bmatrix}$

4. $\begin{bmatrix} 5 & -2 \\ -4 & 6 \\ 7 & 9 \end{bmatrix} + \begin{bmatrix} -11 & 6 \\ 2 & -5 \\ 4 & -7 \end{bmatrix}$

5. $\begin{bmatrix} 8 & 0 & -6 \\ 4 & 5 & -11 \\ -7 & 3 & 4 \end{bmatrix} - \begin{bmatrix} -2 & 1 & 7 \\ 3 & -4 & 3 \\ -8 & 5 & 6 \end{bmatrix}$

6. $\begin{bmatrix} \frac{3}{4} & \frac{2}{5} \\ -\frac{1}{2} & \frac{4}{3} \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{2} \end{bmatrix}$

Lesson 4-2

4-2 Study Guide and Intervention *(continued)*

Operations with Matrices

Scalar Multiplication You can multiply an $m \times n$ matrix by a scalar k.

Scalar Multiplication	$k\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$

Example **If $A = \begin{bmatrix} 4 & 0 \\ -6 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 7 & 8 \end{bmatrix}$, find $3B - 2A$.**

$3B - 2A = 3\begin{bmatrix} -1 & 5 \\ 7 & 8 \end{bmatrix} - 2\begin{bmatrix} 4 & 0 \\ -6 & 3 \end{bmatrix}$ Substitution

$= \begin{bmatrix} 3(-1) & 3(5) \\ 3(7) & 3(8) \end{bmatrix} - \begin{bmatrix} 2(4) & 2(0) \\ 2(-6) & 2(3) \end{bmatrix}$ Multiply.

$= \begin{bmatrix} -3 & 15 \\ 21 & 24 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -12 & 6 \end{bmatrix}$ Simplify.

$= \begin{bmatrix} -3 - 8 & 15 - 0 \\ 21 - (-12) & 24 - 6 \end{bmatrix}$ Subtract.

$= \begin{bmatrix} -11 & 15 \\ 33 & 18 \end{bmatrix}$ Simplify.

Exercises

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1. $6\begin{bmatrix} 2 & -5 & 3 \\ 0 & 7 & -1 \\ -4 & 6 & 9 \end{bmatrix}$

2. $-\frac{1}{3}\begin{bmatrix} 6 & 15 & 9 \\ 51 & -33 & 24 \\ -18 & 3 & 45 \end{bmatrix}$

3. $0.2\begin{bmatrix} 25 & -10 & -45 \\ 5 & 55 & -30 \\ 60 & 35 & -95 \end{bmatrix}$

4. $3\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix} - 2\begin{bmatrix} -1 & 2 \\ -3 & 5 \end{bmatrix}$

5. $-2\begin{bmatrix} 3 & -1 \\ 0 & 7 \end{bmatrix} + 4\begin{bmatrix} -2 & 0 \\ 2 & 5 \end{bmatrix}$

6. $2\begin{bmatrix} 6 & -10 \\ -5 & 8 \end{bmatrix} + 5\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

7. $4\begin{bmatrix} 1 & -2 & 5 \\ -3 & 4 & 1 \end{bmatrix} - 2\begin{bmatrix} 4 & 3 & -4 \\ 2 & -5 & -1 \end{bmatrix}$

8. $8\begin{bmatrix} 2 & 1 \\ 3 & -1 \\ -2 & 4 \end{bmatrix} + 3\begin{bmatrix} 4 & 0 \\ -2 & 3 \\ 3 & -4 \end{bmatrix}$

9. $\frac{1}{4}\left(\begin{bmatrix} 9 & 1 \\ -7 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -5 \\ 1 & 7 \end{bmatrix}\right)$

4-2 Skills Practice

Operations with Matrices

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1. $[5 \quad -4] + [4 \quad 5]$

2. $\begin{bmatrix} 8 & 3 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 0 & -7 \\ 6 & 2 \end{bmatrix}$

3. $[3 \quad 1 \quad 6] + \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

4. $\begin{bmatrix} 5 & -1 & 2 \\ 1 & 8 & -6 \end{bmatrix} + \begin{bmatrix} 9 & 9 & 2 \\ 4 & 6 & 4 \end{bmatrix}$

5. $3[9 \quad 4 \quad -3]$

6. $[6 \quad -3] - 4[4 \quad 7]$

7. $-2\begin{bmatrix} -2 & 5 \\ 5 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

8. $3\begin{bmatrix} 8 \\ 0 \\ -3 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 2 \\ 10 \end{bmatrix}$

9. $5\begin{bmatrix} -4 & 6 \\ 10 & 1 \\ -1 & 1 \end{bmatrix} + 2\begin{bmatrix} 6 & 5 \\ -3 & -2 \\ 1 & 0 \end{bmatrix}$

10. $3\begin{bmatrix} 3 & 1 & 3 \\ -4 & 7 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & -1 & 5 \\ 6 & 6 & -3 \end{bmatrix}$

Use $A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & 4 \\ 3 & 1 \end{bmatrix}$ to find the following.

11. $A + B$

12. $B - C$

13. $B - A$

14. $A + B + C$

15. $3B$

16. $-5C$

17. $A - 4C$

18. $2B + 3A$

NAME ____ DATE ____________ PERIOD _____

4-2 Practice

Operations with Matrices

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1. $\begin{bmatrix} 2 & -1 \\ 3 & 7 \\ 14 & -9 \end{bmatrix} + \begin{bmatrix} -6 & 9 \\ 7 & -11 \\ -8 & 17 \end{bmatrix}$

2. $\begin{bmatrix} 4 \\ -71 \\ 18 \end{bmatrix} - \begin{bmatrix} -67 \\ 45 \\ -24 \end{bmatrix}$

3. $-3\begin{bmatrix} -1 & 0 \\ 17 & -11 \end{bmatrix} + 4\begin{bmatrix} -3 & 16 \\ -21 & 12 \end{bmatrix}$

4. $7\begin{bmatrix} 2 & -1 & 8 \\ 4 & 7 & 9 \end{bmatrix} - 2\begin{bmatrix} -1 & 4 & -3 \\ 7 & 2 & -6 \end{bmatrix}$

5. $-2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 10 \\ 18 \end{bmatrix}$

6. $\frac{3}{4}\begin{bmatrix} 8 & 12 \\ -16 & 20 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} 27 & -9 \\ 54 & -18 \end{bmatrix}$

Use $A = \begin{bmatrix} 4 & -1 & 0 \\ -3 & 6 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 4 & 5 \\ 1 & 0 & -9 \end{bmatrix}$, and $C = \begin{bmatrix} 10 & -8 & 6 \\ -6 & -4 & 20 \end{bmatrix}$ to find the following.

7. $A - B$

8. $A - C$

9. $-3B$

10. $4B - A$

11. $-2B - 3C$

12. $A + 0.5C$

ECONOMICS For Exercises 13 and 14, use the table that shows loans by an economic development board to women and men starting new businesses.

	Women		Men	
	Businesses	Loan Amount ($)	Businesses	Loan Amount ($)
1999	27	$567,000	36	$864,000
2000	41	$902,000	32	$672,000
2001	35	$777,000	28	$562,000

13. Write two matrices that represent the number of new businesses and loan amounts, one for women and one for men.

14. Find the sum of the numbers of new businesses and loan amounts for both men and women over the three-year period expressed as a matrix.

15. PET NUTRITION Use the table that gives nutritional information for two types of dog food. Find the difference in the percent of protein, fat, and fiber between Mix B and Mix A expressed as a matrix.

	% Protein	% Fat	% Fiber
Mix A	22	12	5
Mix B	24	8	8

NAME ____ DATE ____________ PERIOD _____

4-2 Reading to Learn Mathematics

Operations with Matrices

Pre-Activity **How can matrices be used to calculate daily dietary needs?**

Read the introduction to Lesson 4-2 at the top of page 160 in your textbook.

- Write a sum that represents the total number of Calories in the patient's diet for Day 2. (Do not actually calculate the sum.)
- Write the sum that represents the total fat content in the patient's diet for Day 3. (Do not actually calculate the sum.)

Reading the Lesson

1. For each pair of matrices, give the dimensions of the indicated sum, difference, or scalar product. If the indicated sum, difference, or scalar product does not exist, write *impossible*.

$A = \begin{bmatrix} 3 & 5 & 6 \\ -2 & 8 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}$

$C = \begin{bmatrix} 5 & 10 \\ -3 & 6 \\ 4 & 12 \end{bmatrix}$ $\quad D = \begin{bmatrix} -3 & 6 & 0 \\ -8 & 4 & 0 \end{bmatrix}$

$A + D$: ____________ $C + D$: ____________ $5B$: ____________

$-4C$: ____________ $2D - 3A$: ____________

2. Suppose that M, N, and P are nonzero 2×4 matrices and k is a negative real number. Indicate whether each of the following statements is *true* or *false*.

a. $M + (N + P) = M + (P + N)$ **b.** $M - N = N - M$

c. $M - (N - P) = (M - N) - P$ **d.** $k(M - N) = kM - kN$

Helping You Remember

3. The mathematical term *scalar* may be unfamiliar, but its meaning is related to the word *scale* as used in a *scale of miles* on a map. How can this usage of the word *scale* help you remember the meaning of *scalar*?

Lesson 4-2

NAME ______________________________ DATE ____________ PERIOD _____

4-2 Enrichment

Sundaram's Sieve

The properties and patterns of prime numbers have fascinated many mathematicians. In 1934, a young East Indian student named Sundaram constructed the following matrix.

4	7	10	13	16	19	22	25	. . .
7	12	17	22	27	32	37	42	. . .
10	17	24	31	38	45	52	59	. . .
13	22	31	40	49	58	67	76	. . .
16	27	38	49	60	71	82	93	. . .
.	.	.	.	.	.	.	.	. . .

A surprising property of this matrix is that it can be used to determine whether or not some numbers are prime.

Complete these problems to discover this property.

1. The first row and the first column are created by using an arithmetic pattern. What is the common difference used in the pattern?

2. Find the next four numbers in the first row.

3. What are the common differences used to create the patterns in rows 2, 3, 4, and 5?

4. Write the next two rows of the matrix. Include eight numbers in each row.

5. Choose any five numbers from the matrix. For each number n, that you chose from the matrix, find $2n + 1$.

6. Write the factorization of each value of $2n + 1$ that you found in problem 5.

7. Use your results from problems 5 and 6 to complete this statement: If n occurs in the matrix, then $2n + 1$ ______________ (is/is not) a prime number.

8. Choose any five numbers that are not in the matrix. Find $2n + 1$ for each of these numbers. Show that each result is a prime number.

9. Complete this statement: If n does not occur in the matrix, then $2n + 1$ is ________________________.

NAME ______________________________ DATE ____________ PERIOD _____

4-3 Study Guide and Intervention

Multiplying Matrices

Multiply Matrices You can multiply two matrices if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.

Multiplication of Matrices	$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = \begin{bmatrix} a_1x_1 + b_1x_2 & a_1y_1 + b_1y_2 \\ a_2x_1 + b_2x_2 & a_2y_1 + b_2y_2 \end{bmatrix}$

Example **Find AB if $A = \begin{bmatrix} -4 & 3 \\ 2 & -2 \\ 1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$.**

$AB = \begin{bmatrix} -4 & 3 \\ 2 & -2 \\ 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$ Substitution

$= \begin{bmatrix} -4(5) + 3(-1) & -4(-2) + 3(3) \\ 2(5) + (-2)(-1) & 2(-2) + (-2)(3) \\ 1(5) + 7(-1) & 1(-2) + 7(3) \end{bmatrix}$ Multiply columns by rows.

$= \begin{bmatrix} -23 & 17 \\ 12 & -10 \\ -2 & 19 \end{bmatrix}$ Simplify.

Exercises

Find each product, if possible.

1. $\begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 0 \\ 3 & 7 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$

4. $\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & -2 \\ -3 & 1 & 1 \end{bmatrix}$

5. $\begin{bmatrix} 3 & -2 \\ 0 & 4 \\ -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 5 & -2 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 \\ -2 & 5 \end{bmatrix}$

7. $\begin{bmatrix} 6 & 10 \\ -4 & 3 \\ -2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 4 & -3 \end{bmatrix}$

8. $\begin{bmatrix} 7 & -2 \\ 5 & -4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ -2 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & -2 \\ -1 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 \\ 3 & 1 \\ -2 & 4 \end{bmatrix}$

4-3 Study Guide and Intervention *(continued)*

Multiplying Matrices

Multiplicative Properties The Commutative Property of Multiplication does *not* hold for matrices.

Properties of Matrix Multiplication	For any matrices *A*, *B*, and *C* for which the matrix product is defined, and any scalar *c*, the following properties are true.
Associative Property of Matrix Multiplication	$(AB)C = A(BC)$
Associative Property of Scalar Multiplication	$c(AB) = (cA)B = A(cB)$
Left Distributive Property	$C(A + B) = CA + CB$
Right Distributive Property	$(A + B)C = AC + BC$

Example **Use $A = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$ to find each product.**

a. $(A + B)C$

$$(A + B)C = \left(\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix}\right) \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -3 \\ 7 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6(1) + (-3)(6) & 6(-2) + (-3)(3) \\ 7(1) + (-2)(6) & 7(-2) + (-2)(3) \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -21 \\ -5 & -20 \end{bmatrix}$$

b. $AC + BC$

$$AC + BC = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 5 & -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4(1) + (-3)(6) & 4(-2) + (-3)(3) \\ 2(1) + 1(6) & 2(-2) + 1(3) \end{bmatrix} + \begin{bmatrix} 2(1) + 0(6) & 2(-2) + 0(3) \\ 5(1) + (-3)(6) & 5(-2) + (-3)(3) \end{bmatrix}$$

$$= \begin{bmatrix} -14 & -17 \\ 8 & -1 \end{bmatrix} + \begin{bmatrix} 2 & -4 \\ -13 & -19 \end{bmatrix} = \begin{bmatrix} -12 & -21 \\ -5 & -20 \end{bmatrix}$$

Note that although the results in the example illustrate the Right Distributive Property, they do not prove it.

Exercises

Use $A = \begin{bmatrix} 3 & 2 \\ 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 4 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -\frac{1}{2} & -2 \\ 1 & -3 \end{bmatrix}$, and scalar $c = -4$ to determine whether each of the following equations is true for the given matrices.

1. $c(AB) = (cA)B$
2. $AB = BA$
3. $BC = CB$
4. $(AB)C = A(BC)$
5. $C(A + B) = AC + BC$
6. $c(A + B) = cA + cB$

NAME ______________________________ DATE ____________ PERIOD _____

4-3 Skills Practice

Multiplying Matrices

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{2 \times 5} \cdot B_{5 \times 1}$

2. $M_{1 \times 3} \cdot N_{3 \times 2}$

3. $B_{3 \times 2} \cdot A_{3 \times 2}$

4. $R_{4 \times 4} \cdot S_{4 \times 1}$

5. $X_{3 \times 3} \cdot Y_{3 \times 4}$

6. $A_{6 \times 4} \cdot B_{4 \times 5}$

Find each product, if possible.

7. $\begin{bmatrix} 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

8. $\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

10. $\begin{bmatrix} 3 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$

11. $\begin{bmatrix} -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$

12. $\begin{bmatrix} -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 & -2 \end{bmatrix}$

13. $\begin{bmatrix} 5 \\ 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

14. $\begin{bmatrix} 2 & -2 \\ 4 & 5 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$

15. $\begin{bmatrix} -4 & 4 \\ -2 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ 0 & 2 \end{bmatrix}$

16. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

Use $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 5 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$, and scalar $c = 2$ to determine whether the following equations are true for the given matrices.

17. $(AC)c = A(Cc)$

18. $AB = BA$

19. $B(A + C) = AB + BC$

20. $(A - B)c = Ac - Bc$

Lesson 4-3

4-3 Practice

Multiplying Matrices

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{7 \times 4} \cdot B_{4 \times 3}$

2. $A_{3 \times 5} \cdot M_{5 \times 8}$

3. $M_{2 \times 1} \cdot A_{1 \times 6}$

4. $M_{3 \times 2} \cdot A_{3 \times 2}$

5. $P_{1 \times 9} \cdot Q_{9 \times 1}$

6. $P_{9 \times 1} \cdot Q_{1 \times 9}$

Find each product, if possible.

7. $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 \\ 2 & 5 \end{bmatrix}$

9. $\begin{bmatrix} -3 & 0 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix}$

10. $\begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix}$

11. $\begin{bmatrix} 4 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$

12. $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & 2 \end{bmatrix}$

13. $\begin{bmatrix} -6 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

14. $\begin{bmatrix} -15 & -9 \end{bmatrix} \cdot \begin{bmatrix} 6 & 11 \\ 23 & -10 \end{bmatrix}$

Use $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ -2 & -1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, and scalar $c = 3$ to determine whether the following equations are true for the given matrices.

15. $AC = CA$

16. $A(B + C) = BA + CA$

17. $(AB)c = c(AB)$

18. $(A + C)B = B(A + C)$

RENTALS For Exercises 19–21, use the following information.

For their one-week vacation, the Montoyas can rent a 2-bedroom condominium for \$1796, a 3-bedroom condominium for \$2165, or a 4-bedroom condominium for \$2538. The table shows the number of units in each of three complexes.

	2-Bedroom	3-Bedroom	4-Bedroom
Sun Haven	36	24	22
Surfside	29	32	42
Seabreeze	18	22	18

19. Write a matrix that represents the number of each type of unit available at each complex and a matrix that represents the weekly charge for each type of unit.

20. If all of the units in the three complexes are rented for the week at the rates given the Montoyas, express the income of each of the three complexes as a matrix.

21. What is the total income of all three complexes for the week?

4-3 Reading to Learn Mathematics

Multiplying Matrices

Pre-Activity **How can matrices be used in sports statistics?**

Read the introduction to Lesson 4-3 at the top of page 167 in your textbook.

Write a sum that shows the total points scored by the Oakland Raiders during the 2000 season. (The sum will include multiplications. Do not actually calculate this sum.)

Reading the Lesson

1. Determine whether each indicated matrix product is defined. If so, state the dimensions of the product. If not, write *undefined*.

a. $M_{3 \times 2}$ and $N_{2 \times 3}$ MN: ______________ NM: ______________

b. $M_{1 \times 2}$ and $N_{1 \times 2}$ MN: ______________ NM: ______________

c. $M_{4 \times 1}$ and $N_{1 \times 4}$ MN: ______________ NM: ______________

d. $M_{3 \times 4}$ and $N_{4 \times 4}$ MN: ______________ NM: ______________

2. The regional sales manager for a chain of computer stores wants to compare the revenue from sales of one model of notebook computer and one model of printer for three stores in his area. The notebook computer sells for $1850 and the printer for $175. The number of computers and printers sold at the three stores during September are shown in the following table.

Store	Computers	Printers
A	128	101
B	205	166
C	97	73

Write a matrix product that the manager could use to find the total revenue for computers and printers for each of the three stores. (Do not calculate the product.)

Helping You Remember

3. Many students find the procedure of matrix multiplication confusing at first because it is unfamiliar. Think of an easy way to use the letters R and C to remember how to multiply matrices and what the dimensions of the product will be.

4-3 Enrichment

Fourth-Order Determinants

To find the value of a 4 × 4 determinant, use a method called **expansion by minors**.

First write the expansion. Use the first row of the determinant.
Remember that the signs of the terms alternate.

$$\begin{vmatrix} 6 & -3 & 2 & 7 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & 1 & -4 \\ 6 & 0 & -2 & 0 \end{vmatrix} = 6\begin{vmatrix} 4 & 3 & 5 \\ 2 & 1 & -4 \\ 0 & -2 & 0 \end{vmatrix} - (-3)\begin{vmatrix} 0 & 3 & 5 \\ 0 & 1 & -4 \\ 6 & -2 & 0 \end{vmatrix} + 2\begin{vmatrix} 0 & 4 & 5 \\ 0 & 2 & -4 \\ 6 & 0 & 0 \end{vmatrix} - 7\begin{vmatrix} 0 & 4 & 3 \\ 0 & 2 & 1 \\ 6 & 0 & -2 \end{vmatrix}$$

Then evaluate each 3 × 3 determinant. Use any row.

$$\begin{vmatrix} 4 & 3 & 5 \\ 2 & 1 & -4 \\ 0 & -2 & 0 \end{vmatrix} = -(-2)\begin{vmatrix} 4 & 5 \\ 2 & -4 \end{vmatrix}$$
$$= 2(-16 - 10)$$
$$= -52$$

$$\begin{vmatrix} 0 & 3 & 5 \\ 0 & 1 & -4 \\ 6 & -2 & 0 \end{vmatrix} = -3\begin{vmatrix} 0 & -4 \\ 6 & 0 \end{vmatrix} + 5\begin{vmatrix} 0 & 1 \\ 6 & -2 \end{vmatrix}$$
$$= -3(24) + 5(-6)$$
$$= -102$$

$$\begin{vmatrix} 0 & 4 & 5 \\ 0 & 2 & -4 \\ 6 & 0 & 0 \end{vmatrix} = 6\begin{vmatrix} 4 & 5 \\ 2 & -4 \end{vmatrix}$$
$$= 6(-16 - 10)$$
$$= -156$$

$$\begin{vmatrix} 0 & 4 & 3 \\ 0 & 2 & 1 \\ 6 & 0 & -2 \end{vmatrix} = -4\begin{vmatrix} 0 & 1 \\ 6 & -2 \end{vmatrix} + 3\begin{vmatrix} 0 & 2 \\ 6 & 0 \end{vmatrix}$$
$$= -4(-6) + 3(-12)$$
$$= -12$$

Finally, evaluate the original 4 × 4 determinant.

$$\begin{vmatrix} 6 & -3 & 2 & 7 \\ 0 & 4 & 3 & 5 \\ 0 & 2 & 1 & -4 \\ 6 & 0 & -2 & 0 \end{vmatrix} = 6(-52) + 3(-102) + 2(-156) - 7(-12) = -846$$

Evaluate each determinant.

1. $\begin{vmatrix} 1 & 2 & 3 & 1 \\ 4 & 3 & -1 & 0 \\ 2 & -5 & 4 & 4 \\ 1 & -2 & 0 & 2 \end{vmatrix}$

2. $\begin{vmatrix} 3 & 3 & 3 & 3 \\ 2 & 1 & 2 & 1 \\ 4 & 3 & -1 & 5 \\ 2 & 5 & 0 & 1 \end{vmatrix}$

3. $\begin{vmatrix} 1 & 4 & 3 & 0 \\ -2 & -3 & 6 & 4 \\ 5 & 1 & 1 & 2 \\ 4 & 2 & 5 & -1 \end{vmatrix}$

NAME _______________________________ DATE ____________ PERIOD _____

4-4 Study Guide and Intervention

Transformations with Matrices

Translations and Dilations Matrices that represent coordinates of points on a plane are useful in describing transformations.

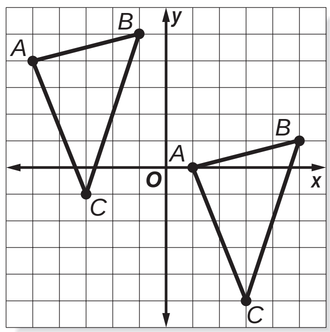
Translation	a transformation that moves a figure from one location to another on the coordinate plane

You can use matrix addition and a translation matrix to find the coordinates of the translated figure.

Dilation	a transformation in which a figure is enlarged or reduced

You can use scalar multiplication to perform dilations.

Example **Find the coordinates of the vertices of the image of $\triangle ABC$ with vertices $A(-5, 4)$, $B(-1, 5)$, and $C(-3, -1)$ if it is moved 6 units to the right and 4 units down. Then graph $\triangle ABC$ and its image $\triangle A'B'C'$.**



Write the vertex matrix for $\triangle ABC$. $\begin{bmatrix} -5 & -1 & -3 \\ 4 & 5 & -1 \end{bmatrix}$

Add the translation matrix $\begin{bmatrix} 6 & 6 & 6 \\ -4 & -4 & -4 \end{bmatrix}$ to the vertex matrix of $\triangle ABC$.

$$\begin{bmatrix} -5 & -1 & -3 \\ 4 & 5 & -1 \end{bmatrix} + \begin{bmatrix} 6 & 6 & 6 \\ -4 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 3 \\ 0 & 1 & -5 \end{bmatrix}$$

The coordinates of the vertices of $\triangle A'B'C'$ are $A'(1, 0)$, $B'(5, 1)$, and $C'(3, -5)$.

Exercises

For Exercises 1 and 2 use the following information. Quadrilateral *QUAD* with vertices $Q(-1, -3)$, $U(0, 0)$, $A(5, -1)$, and $D(2, -5)$ is translated 3 units to the left and 2 units up.

1. Write the translation matrix.

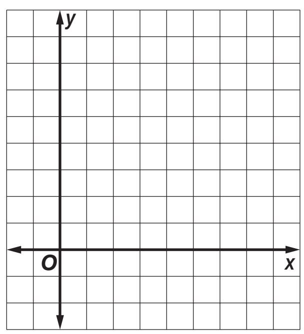
2. Find the coordinates of the vertices of $Q'U'A'D'$.

For Exercises 3–5, use the following information. The vertices of $\triangle ABC$ are $A(4, -2)$, $B(2, 8)$, and $C(8, 2)$. The triangle is dilated so that its perimeter is one-fourth the original perimeter.

3. Write the coordinates of the vertices of $\triangle ABC$ in a vertex matrix.

4. Find the coordinates of the vertices of image $\triangle A'B'C'$.

5. Graph the preimage and the image.



Lesson 4-4

4-4 Study Guide and Intervention *(continued)*

Transformations with Matrices

Reflections and Rotations

Reflection Matrices	For a reflection over the:	x-axis	y-axis	line $y = x$
	multiply the vertex matrix on the left by:	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Rotation Matrices	For a counterclockwise rotation about the origin of:	90°	180°	270°
	multiply the vertex matrix on the left by:	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Example **Find the coordinates of the vertices of the image of $\triangle ABC$ with $A(3, 5)$, $B(-2, 4)$, and $C(1, -1)$ after a reflection over the line $y = x$.**

Write the ordered pairs as a vertex matrix. Then multiply the vertex matrix by the reflection matrix for $y = x$.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 1 \\ 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & -1 \\ 3 & -2 & 1 \end{bmatrix}$$

The coordinates of the vertices of $A'B'C'$ are $A'(5, 3)$, $B'(4, -2)$, and $C'(-1, 1)$.

Exercises

1. The coordinates of the vertices of quadrilateral $ABCD$ are $A(-2, 1)$, $B(-1, 3)$, $C(2, 2)$, and $D(2, -1)$. What are the coordinates of the vertices of the image $A'B'C'D'$ after a reflection over the y-axis?

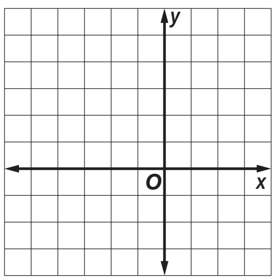
2. Triangle DEF with vertices $D(-2, 5)$, $E(1, 4)$, and $F(0, -1)$ is rotated 90° counterclockwise about the origin.

a. Write the coordinates of the triangle in a vertex matrix.

b. Write the rotation matrix for this situation.

c. Find the coordinates of the vertices of $\triangle D'E'F'$.

d. Graph $\triangle DEF$ and $\triangle D'E'F'$.



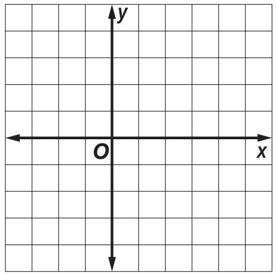
NAME ______________________________ DATE ____________ PERIOD _____

4-4 Skills Practice

Transformations with Matrices

For Exercises 1–3, use the following information.

Triangle *ABC* with vertices $A(2, 3)$, $B(0, 4)$, and $C(-3, -3)$ is translated 3 units right and 1 unit down.



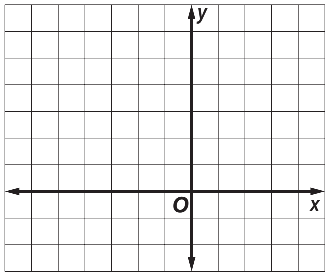
1. Write the translation matrix.

2. Find the coordinates of $\triangle A'B'C'$.

3. Graph the preimage and the image.

For Exercises 4–6, use the following information.

The vertices of $\triangle RST$ are $R(-3, 1)$, $S(2, -1)$, and $T(1, 3)$. The triangle is dilated so that its perimeter is twice the original perimeter.



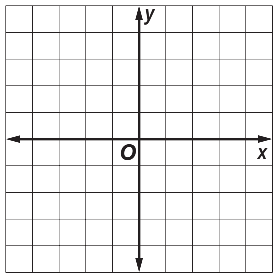
4. Write the coordinates of $\triangle RST$ in a vertex matrix.

5. Find the coordinates of the image $\triangle R'S'T'$.

6. Graph $\triangle RST$ and $\triangle R'S'T'$.

For Exercises 7–10, use the following information.

The vertices of $\triangle DEF$ are $D(4, 0)$, $E(0, -1)$, and $F(2, 3)$. The triangle is reflected over the *x*-axis.



7. Write the coordinates of $\triangle DEF$ in a vertex matrix.

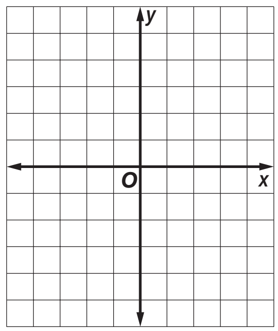
8. Write the reflection matrix for this situation.

9. Find the coordinates of $\triangle D'E'F'$.

10. Graph $\triangle DEF$ and $\triangle D'E'F'$.

For Exercises 11–14, use the following information.

Triangle *XYZ* with vertices $X(1, -3)$, $Y(-4, 1)$, and $Z(-2, 5)$ is rotated 180° counterclockwise about the origin.



11. Write the coordinates of the triangle in a vertex matrix.

12. Write the rotation matrix for this situation.

13. Find the coordinates of $\triangle X'Y'Z'$.

14. Graph the preimage and the image.

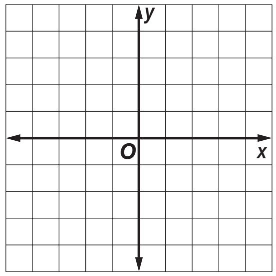
NAME ______________________________ DATE __________ PERIOD _____

4-4 Practice

Transformations with Matrices

For Exercises 1–3, use the following information.

Quadrilateral *WXYZ* with vertices $W(-3, 2)$, $X(-2, 4)$, $Y(4, 1)$, and $Z(3, 0)$ is translated 1 unit left and 3 units down.



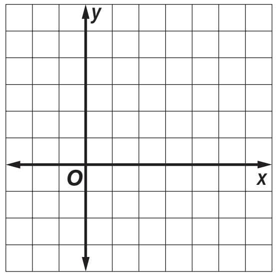
1. Write the translation matrix.

2. Find the coordinates of quadrilateral $W'X'Y'Z'$.

3. Graph the preimage and the image.

For Exercises 4–6, use the following information.

The vertices of $\triangle RST$ are $R(6, 2)$, $S(3, -3)$, and $T(-2, 5)$. The triangle is dilated so that its perimeter is one half the original perimeter.



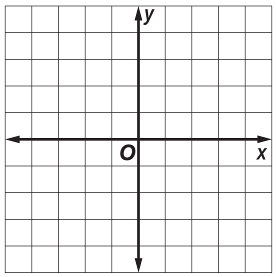
4. Write the coordinates of $\triangle RST$ in a vertex matrix.

5. Find the coordinates of the image $\triangle R'S'T'$.

6. Graph $\triangle RST$ and $\triangle R'S'T'$.

For Exercises 7–10, use the following information.

The vertices of quadrilateral *ABCD* are $A(-3, 2)$, $B(0, 3)$, $C(4, -4)$, and $D(-2, -2)$. The quadrilateral is reflected over the *y*-axis.



7. Write the coordinates of *ABCD* in a vertex matrix.

8. Write the reflection matrix for this situation.

9. Find the coordinates of $A'B'C'D'$.

10. Graph *ABCD* and $A'B'C'D'$.

11. ARCHITECTURE Using architectural design software, the Bradleys plot their kitchen plans on a grid with each unit representing 1 foot. They place the corners of an island at (2, 8), (8, 11), (3, 5), and (9, 8). If the Bradleys wish to move the island 1.5 feet to the right and 2 feet down, what will the new coordinates of its corners be?

12. BUSINESS The design of a business logo calls for locating the vertices of a triangle at (1.5, 5), (4, 1), and (1, 0) on a grid. If design changes require rotating the triangle 90º counterclockwise, what will the new coordinates of the vertices be?

4-4 Reading to Learn Mathematics

Transformations with Matrices

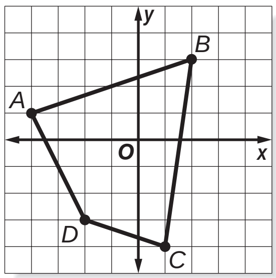
Pre-Activity **How are transformations used in computer animation?**

Read the introduction to Lesson 4-4 at the top of page 175 in your textbook.

Describe how you can change the orientation of a figure without changing its size or shape.

Reading the Lesson

1. a. Write the vertex matrix for the quadrilateral *ABCD* shown in the graph at the right.



b. Write the vertex matrix that represents the position of the quadrilateral $A'B'C'D'$ that results when quadrilateral *ABCD* is translated 3 units to the right and 2 units down.

2. Describe the transformation that corresponds to each of the following matrices.

a. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 3 & 3 \\ -4 & -4 & -4 \end{bmatrix}$

c. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Helping You Remember

3. Describe a way to remember which of the reflection matrices corresponds to reflection over the x-axis.

Lesson 4-4

4-4 Enrichment

Properties of Determinants

The following properties often help when evaluating determinants.

- If all the elements of a row (or column) are zero, the value of the determinant is zero.

  $\begin{vmatrix} a & b \\ 0 & 0 \end{vmatrix} = 0$

- Multiplying all the elements of a row (or column) by a constant is equivalent to multiplying the value of the determinant by the constant.

  $3\begin{vmatrix} 4 & -1 \\ 5 & 3 \end{vmatrix} = \begin{vmatrix} 12 & -3 \\ 5 & 3 \end{vmatrix}$

- If two rows (or columns) have equal corresponding elements, the value of the determinant is zero.

  $\begin{vmatrix} 5 & 5 \\ -3 & -3 \end{vmatrix} = 0$

- The value of a determinant is unchanged if any multiple of a row (or column) is added to corresponding elements of another row (or column).

  $\begin{vmatrix} 4 & -3 \\ 2 & 5 \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 2 & 5 \end{vmatrix}$

  (Row 2 is added to row 1.)

- If two rows (or columns) are interchanged, the sign of the determinant is changed.

  $\begin{vmatrix} 4 & 5 \\ -3 & 8 \end{vmatrix} = \begin{vmatrix} -3 & 8 \\ 5 & 5 \end{vmatrix}$

- The value of the determinant is unchanged if row 1 is interchanged with column 1, and row 2 is interchanged with column 2. The result is called the transpose.

  $\begin{vmatrix} 5 & -7 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 5 & 3 \\ -7 & 4 \end{vmatrix}$

Exercises 1–6

Verify each property above by evaluating the given determinants and give another example of the property.

NAME ______________________________ DATE ____________ PERIOD _____

4-5 Study Guide and Intervention

Determinants

Determinants of 2 × 2 Matrices

Second-Order Determinant	For the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Example **Find the value of each determinant.**

a. $\begin{vmatrix} 6 & 3 \\ -8 & 5 \end{vmatrix}$

$\begin{vmatrix} 6 & 3 \\ -8 & 5 \end{vmatrix} = 6(5) - 3(-8)$

$= 30 - (-24)$ or 54

b. $\begin{vmatrix} 11 & -5 \\ 9 & 3 \end{vmatrix}$

$\begin{vmatrix} 11 & -5 \\ 9 & 3 \end{vmatrix} = 11(-3) - (-5)(9)$

$= -33 - (-45)$ or 12

Exercises

Find the value of each determinant.

1. $\begin{vmatrix} 6 & -2 \\ 5 & 7 \end{vmatrix}$

2. $\begin{vmatrix} -8 & 3 \\ -2 & 1 \end{vmatrix}$

3. $\begin{vmatrix} 3 & 9 \\ 4 & 6 \end{vmatrix}$

4. $\begin{vmatrix} 5 & 12 \\ -7 & -4 \end{vmatrix}$

5. $\begin{vmatrix} -6 & -3 \\ -4 & -1 \end{vmatrix}$

6. $\begin{vmatrix} 4 & 7 \\ 5 & 9 \end{vmatrix}$

7. $\begin{vmatrix} 14 & 8 \\ 9 & -3 \end{vmatrix}$

8. $\begin{vmatrix} 15 & 12 \\ 23 & 28 \end{vmatrix}$

9. $\begin{vmatrix} -8 & 35 \\ 5 & 20 \end{vmatrix}$

10. $\begin{vmatrix} 10 & 16 \\ 22 & 40 \end{vmatrix}$

11. $\begin{vmatrix} 24 & -8 \\ 7 & -3 \end{vmatrix}$

12. $\begin{vmatrix} 13 & 62 \\ -4 & 19 \end{vmatrix}$

13. $\begin{vmatrix} 0.2 & 8 \\ -1.5 & 15 \end{vmatrix}$

14. $\begin{vmatrix} 8.6 & 0.5 \\ 14 & 5 \end{vmatrix}$

15. $\begin{vmatrix} 20 & 110 \\ 0.1 & 1.4 \end{vmatrix}$

16. $\begin{vmatrix} 4.8 & 2.1 \\ 3.4 & 5.3 \end{vmatrix}$

17. $\begin{vmatrix} \frac{2}{3} & -\frac{1}{2} \\ \frac{1}{6} & \frac{1}{5} \end{vmatrix}$

18. $\begin{vmatrix} 6.8 & 15 \\ -0.2 & 5 \end{vmatrix}$

Lesson 4-5

4-5 Study Guide and Intervention *(continued)*

Determinants

Determinants of 3 × 3 Matrices

Third-Order Determinants	$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$

Area of a Triangle	The area of a triangle having vertices (a, b), (c, d) and (e, f) is $\lvert A \rvert$, where $A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$.

Example **Evaluate** $\begin{vmatrix} 4 & 5 & -2 \\ 1 & 3 & 0 \\ 2 & -3 & 6 \end{vmatrix}$.

$$\begin{vmatrix} 4 & 5 & -2 \\ 1 & 3 & 0 \\ 2 & -3 & 6 \end{vmatrix} = 4\begin{vmatrix} 3 & 0 \\ -3 & 6 \end{vmatrix} - 5\begin{vmatrix} 1 & 0 \\ 2 & 6 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} \quad \text{Third-order determinant}$$

$$= 4(18 - 0) - 5(6 - 0) - 2(-3 - 6) \quad \text{Evaluate } 2 \times 2 \text{ determinants.}$$

$$= 4(18) - 5(6) - 2(-9) \quad \text{Simplify.}$$

$$= 72 - 30 + 18 \quad \text{Multiply.}$$

$$= 60 \quad \text{Simplify.}$$

Exercises

Evaluate each determinant.

1. $\begin{vmatrix} 3 & -2 & -2 \\ 0 & 4 & 1 \\ -1 & 5 & -3 \end{vmatrix}$

2. $\begin{vmatrix} 4 & 1 & 0 \\ -2 & 3 & 1 \\ 2 & -2 & 5 \end{vmatrix}$

3. $\begin{vmatrix} 6 & 1 & 4 \\ -2 & 3 & 0 \\ -1 & 3 & 2 \end{vmatrix}$

4. $\begin{vmatrix} 5 & -2 & 2 \\ 3 & 0 & -2 \\ 2 & 4 & -3 \end{vmatrix}$

5. $\begin{vmatrix} 6 & 1 & -4 \\ 3 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix}$

6. $\begin{vmatrix} 5 & -4 & 1 \\ 2 & 3 & -2 \\ -1 & 6 & -3 \end{vmatrix}$

7. Find the area of a triangle with vertices $X(2, -3)$, $Y(7, 4)$, and $Z(-5, 5)$.

NAME ____ DATE ____________ PERIOD _____

4-5 Skills Practice

Determinants

Find the value of each determinant.

1. $\begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix}$

2. $\begin{vmatrix} 10 & 9 \\ 5 & 8 \end{vmatrix}$

3. $\begin{vmatrix} 1 & 6 \\ 1 & 7 \end{vmatrix}$

4. $\begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix}$

5. $\begin{vmatrix} 0 & 9 \\ 5 & 8 \end{vmatrix}$

6. $\begin{vmatrix} 3 & 12 \\ 2 & 8 \end{vmatrix}$

7. $\begin{vmatrix} -5 & 2 \\ 8 & -6 \end{vmatrix}$

8. $\begin{vmatrix} -3 & 1 \\ 8 & -7 \end{vmatrix}$

9. $\begin{vmatrix} 9 & -2 \\ -4 & 1 \end{vmatrix}$

10. $\begin{vmatrix} 1 & -5 \\ 1 & 6 \end{vmatrix}$

11. $\begin{vmatrix} 1 & -3 \\ -3 & 4 \end{vmatrix}$

12. $\begin{vmatrix} -12 & 4 \\ 1 & 4 \end{vmatrix}$

13. $\begin{vmatrix} 3 & -5 \\ 6 & -11 \end{vmatrix}$

14. $\begin{vmatrix} -1 & -3 \\ 5 & -2 \end{vmatrix}$

15. $\begin{vmatrix} -1 & -14 \\ 5 & 2 \end{vmatrix}$

16. $\begin{vmatrix} -1 & 2 \\ 0 & 4 \end{vmatrix}$

17. $\begin{vmatrix} 2 & 2 \\ -1 & 4 \end{vmatrix}$

18. $\begin{vmatrix} -1 & 6 \\ 2 & 5 \end{vmatrix}$

Evaluate each determinant using expansion by minors.

19. $\begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 2 & 3 & -2 \end{vmatrix}$

20. $\begin{vmatrix} 6 & -1 & 1 \\ 5 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix}$

21. $\begin{vmatrix} 2 & 6 & 1 \\ 3 & 5 & -1 \\ 2 & 1 & -2 \end{vmatrix}$

Evaluate each determinant using diagonals.

22. $\begin{vmatrix} 2 & -1 & 6 \\ 3 & 2 & 5 \\ 2 & 3 & 1 \end{vmatrix}$

23. $\begin{vmatrix} 3 & -1 & 2 \\ 1 & 0 & 4 \\ 3 & -2 & 0 \end{vmatrix}$

24. $\begin{vmatrix} 3 & 2 & 2 \\ 1 & -1 & 4 \\ 3 & -1 & 0 \end{vmatrix}$

NAME ______________________________ DATE ____________ PERIOD _____

4-5 Practice

Determinants

Find the value of each determinant.

1. $\begin{vmatrix} 1 & 6 \\ 2 & 7 \end{vmatrix}$

2. $\begin{vmatrix} 9 & 6 \\ 3 & 2 \end{vmatrix}$

3. $\begin{vmatrix} 4 & 1 \\ -2 & -5 \end{vmatrix}$

4. $\begin{vmatrix} -14 & -3 \\ 2 & -2 \end{vmatrix}$

5. $\begin{vmatrix} 4 & -3 \\ -12 & 4 \end{vmatrix}$

6. $\begin{vmatrix} 2 & -5 \\ 5 & -11 \end{vmatrix}$

7. $\begin{vmatrix} 4 & 0 \\ -2 & 9 \end{vmatrix}$

8. $\begin{vmatrix} 3 & -4 \\ 7 & 9 \end{vmatrix}$

9. $\begin{vmatrix} -1 & -11 \\ 10 & -2 \end{vmatrix}$

10. $\begin{vmatrix} 3 & -4 \\ 3.75 & 5 \end{vmatrix}$

11. $\begin{vmatrix} 2 & -1 \\ 3 & -9.5 \end{vmatrix}$

12. $\begin{vmatrix} 0.5 & -0.7 \\ 0.4 & -0.3 \end{vmatrix}$

Evaluate each determinant using expansion by minors.

13. $\begin{vmatrix} -2 & 3 & 1 \\ 0 & 4 & -3 \\ 2 & 5 & -1 \end{vmatrix}$

14. $\begin{vmatrix} 2 & -4 & 1 \\ 3 & 0 & 9 \\ -1 & 5 & 7 \end{vmatrix}$

15. $\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 1 & 1 & -1 \end{vmatrix}$

16. $\begin{vmatrix} 0 & -4 & 0 \\ 2 & -1 & 1 \\ 3 & -2 & 5 \end{vmatrix}$

17. $\begin{vmatrix} 2 & 7 & -6 \\ 8 & 4 & 0 \\ 1 & -1 & 3 \end{vmatrix}$

18. $\begin{vmatrix} -12 & 0 & 3 \\ 7 & 5 & -1 \\ 4 & 2 & -6 \end{vmatrix}$

Evaluate each determinant using diagonals.

19. $\begin{vmatrix} -4 & 3 & -1 \\ 2 & 1 & -2 \\ 4 & 1 & -4 \end{vmatrix}$

20. $\begin{vmatrix} 2 & 2 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{vmatrix}$

21. $\begin{vmatrix} 1 & -4 & -1 \\ 1 & -6 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

22. $\begin{vmatrix} 1 & 2 & -4 \\ 1 & 4 & -6 \\ 2 & 3 & 3 \end{vmatrix}$

23. $\begin{vmatrix} 2 & -1 & -2 \\ 4 & 0 & -2 \\ 0 & 3 & 2 \end{vmatrix}$

24. $\begin{vmatrix} 2 & 1 & 3 \\ 1 & 8 & 0 \\ 0 & 5 & -1 \end{vmatrix}$

25. GEOMETRY Find the area of a triangle whose vertices have coordinates (3, 5), (6, −5), and (−4, 10).

26. LAND MANAGEMENT A fish and wildlife management organization uses a GIS (geographic information system) to store and analyze data for the parcels of land it manages. All of the parcels are mapped on a grid in which 1 unit represents 1 acre. If the coordinates of the corners of a parcel are (−8, 10), (6, 17), and (2, −4), how many acres is the parcel?

NAME ____ DATE ____________ PERIOD _____

4-5 Reading to Learn Mathematics

Determinants

Pre-Activity **How are determinants used to find areas of polygons?**

Read the introduction to Lesson 4-5 at the top of page 182 in your textbook.

In this lesson, you will learn how to find the area of a triangle if you know the coordinates of its vertices using determinants. Describe a method you already know for finding the area of the Bermuda Triangle.

Reading the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. Every matrix has a determinant.

b. If both rows of a 2×2 matrix are identical, the determinant of the matrix will be 0.

c. Every element of a 3×3 matrix has a minor.

d. In order to evaluate a third-order determinant by expansion by minors it is necessary to find the minor of every element of the matrix.

e. If you evaluate a third-order determinant by expansion about the second row, the position signs you will use are $- + -$.

2. Suppose that triangle RST has vertices $R(-2, 5)$, $S(4, 1)$, and $T(0, 6)$.

a. Write a determinant that could be used in finding the area of triangle RST.

b. Explain how you would use the determinant you wrote in part **a** to find the area of the triangle.

Helping You Remember

3. A good way to remember a complicated procedure is to break it down into steps. Write a list of steps for evaluating a third-order determinant using expansion by minors.

Lesson 4-5

4-5 Enrichment

Matrix Multiplication

A furniture manufacturer makes upholstered chairs and wood tables. Matrix A shows the number of hours spent on each item by three different workers. One day the factory receives an order for 10 chairs and 3 tables. This is shown in matrix B.

	woodworker (hours)	finsher (hours)	upholsterer (hours)
chair	4	2	12
table	18	15	0

$$\begin{bmatrix} 4 & 2 & 12 \\ 18 & 15 & 0 \end{bmatrix} = A \qquad \text{number ordered } \begin{bmatrix} 10 & 3 \end{bmatrix} = B \text{ (chair, table)}$$

$$\begin{bmatrix} 10 & 3 \end{bmatrix}\begin{bmatrix} 4 & 2 & 12 \\ 18 & 15 & 0 \end{bmatrix} = [10(4) + 3(18) \quad 10(2) + 3(15) \quad 10(12) + 3(0)] = [94 \quad 65 \quad 120]$$

The product of the two matrices shows the number of hours needed for each type of worker to complete the order: 94 hours for woodworking, 65 hours for finishing, and 120 hours for upholstering.

To find the total labor cost, multiply by a matrix that shows the hourly rate for each worker: $15 for woodworking, $9 for finishing, and $12 for upholstering.

$$C = \begin{bmatrix} 15 \\ 9 \\ 12 \end{bmatrix}[94 \quad 65 \quad 120] = [94(15) + 65(9) + 120(12)] = \$3435$$

Use matrix multiplication to solve these problems.

A candy company packages caramels, chocolates, and hard candy in three different assortments: traditional, deluxe, and superb. For each type of candy the table below gives the number in each assortment, the number of Calories per piece, and the cost to make each piece.

	traditional	deluxe	superb	Calories per piece	cost per piece (cents)
caramels	10	16	15	60	10
chocolates	12	8	25	70	12
card candy	10	16	8	55	6

The company receives an order for 300 traditional, 180 deluxe and 100 superb assortments.

1. Find the number of each type of candy needed to fill the order.

2. Find the total number of Calories in each type of assortment.

3. Find the cost of production for each type of assortment.

4. Find the cost to fill the order.

4-6 Study Guide and Intervention

Cramer's Rule

Systems of Two Linear Equations Determinants provide a way for solving systems of equations.

Cramer's Rule for Two-Variable Systems	The solution of the linear system of equations $ax + by = e$, $cx + dy = f$ is (x, y) where $x = \dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$, $y = \dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$, and $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$.

Example **Use Cramer's Rule to solve the system of equations.** $\mathbf{5x - 10y = 8}$, $\mathbf{10x + 25y = -2}$

$x = \dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$ Cramer's Rule $y = \dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

$= \dfrac{\begin{vmatrix} 8 & -10 \\ -2 & 25 \end{vmatrix}}{\begin{vmatrix} 5 & -10 \\ 10 & 25 \end{vmatrix}}$ $a = 5, b = -10, c = 10, d = 25, e = 8, f = -2$ $= \dfrac{\begin{vmatrix} 5 & 8 \\ 10 & -2 \end{vmatrix}}{\begin{vmatrix} 5 & -10 \\ 10 & 25 \end{vmatrix}}$

$= \dfrac{8(25) - (-2)(-10)}{5(25) - (-10)(10)}$ Evaluate each determinant. $= \dfrac{5(-2) - 8(10)}{5(25) - (-10)(10)}$

$= \dfrac{180}{225}$ or $\dfrac{4}{5}$ Simplify. $= -\dfrac{90}{225}$ or $-\dfrac{2}{5}$

The solution is $\left(\dfrac{4}{5}, -\dfrac{2}{5}\right)$.

Exercises

Use Cramer's Rule to solve each system of equations.

1. $3x - 2y = 7$
$2x + 7y = 38$

2. $x - 4y = 17$
$3x - y = 29$

3. $2x - y = -2$
$4x - y = 4$

4. $2x - y = 1$
$5x + 2y = -29$

5. $4x + 2y = 1$
$5x - 4y = 24$

6. $6x - 3y = -3$
$2x + y = 21$

7. $2x + 7y = 16$
$x - 2y = 30$

8. $2x - 3y = -2$
$3x - 4y = 9$

9. $\dfrac{x}{3} + \dfrac{y}{5} = 2$
$\dfrac{x}{4} - \dfrac{y}{6} = -8$

10. $6x - 9y = -1$
$3x + 18y = 12$

11. $3x - 12y = -14$
$9x + 6y = -7$

12. $8x + 2y = \dfrac{3}{7}$
$5x - 4y = -\dfrac{27}{7}$

4-6 Study Guide and Intervention *(continued)*

Cramer's Rule

Systems of Three Linear Equations

Cramer's Rule for Three-Variable Systems	The solution of the system whose equations are $ax + by + cz = j$, $dx + ey + fz = k$, $gx + hy + iz = l$ is (x, y, z) where $x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$, $y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$, and $z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$ and $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq 0$.

Example **Use Cramer's rule to solve the system of equations.**

$\mathbf{6x + 4y + z = 5}$
$\mathbf{2x + 3y - 2z = -2}$
$\mathbf{8x - 2y + 2z = 10}$

Use the coefficients and constants from the equations to form the determinants. Then evaluate each determinant.

$$x = \frac{\begin{vmatrix} 5 & 4 & 1 \\ -2 & 3 & -2 \\ 10 & -2 & 2 \end{vmatrix}}{\begin{vmatrix} 6 & 4 & 1 \\ 2 & 3 & -2 \\ 8 & -2 & 2 \end{vmatrix}} = \frac{-80}{-96} \text{ or } \frac{5}{6}$$

$$y = \frac{\begin{vmatrix} 6 & 5 & 1 \\ 2 & -2 & -2 \\ 8 & 10 & 2 \end{vmatrix}}{\begin{vmatrix} 6 & 4 & 1 \\ 2 & 3 & -2 \\ 8 & -2 & 2 \end{vmatrix}} = \frac{32}{-96} \text{ or } -\frac{1}{3}$$

$$z = \frac{\begin{vmatrix} 6 & 4 & 5 \\ 2 & 3 & -2 \\ 8 & -2 & 10 \end{vmatrix}}{\begin{vmatrix} 6 & 4 & 1 \\ 2 & 3 & -2 \\ 8 & -2 & 2 \end{vmatrix}} = \frac{-128}{-96} \text{ or } \frac{4}{3}$$

The solution is $\left(\frac{5}{6}, -\frac{1}{3}, \frac{4}{3}\right)$.

Exercises

Use Cramer's rule to solve each system of equations.

1. $x - 2y + 3z = 6$
$2x - y - z = -3$
$x + y + z = 6$

2. $3x + y - 2z = -2$
$4x - 2y - 5z = 7$
$x + y + z = 1$

3. $x - 3y + z = 1$
$2x + 2y - z = -8$
$4x + 7y + 2z = 11$

4. $2x - y + 3z = -5$
$x + y - 5z = 21$
$3x - 2y - 4z = 6$

5. $3x + y - 4z = 7$
$2x - y + 5z = -24$
$10x + 3y - 2z = -2$

6. $2x - y + 4z = 9$
$3x - 2y - 5z = -13$
$x + y - 7z = 0$

4-6 Skills Practice

Cramer's Rule

Use Cramer's Rule to solve each system of equations.

1. $2a + 3b = 6$
$2a + b = -2$

2. $3x + y = 2$
$2x - y = 3$

3. $2m + 3n = -6$
$m - 3n = 6$

4. $x - y = 2$
$2x + 3y = 9$

5. $2x + y = 4$
$7x - 2y = 3$

6. $3r - s = 7$
$5r - 2s = 8$

7. $4g + 5h = 1$
$g + 3h = 2$

8. $7x + 5y = -8$
$9x + 2y = 3$

9. $3x - 4y = 2$
$4x - 3y = 12$

10. $2x - y = 5$
$3x + y = 5$

11. $3p - 6q = 18$
$2p + 3q = 5$

12. $x - 2y = -1$
$2x + y = 3$

13. $5c + 3d = 5$
$2c + 9d = 2$

14. $5t + 2v = 2$
$2t + 3v = -8$

15. $5a - 2b = 14$
$3a + 4b = 11$

16. $65w - 8z = 83$
$9w + 4z = 0$

17. GEOMETRY The two sides of an angle are contained in the lines whose equations are $3x + 2y = 4$ and $x - 3y = 5$. Find the coordinates of the vertex of the angle.

Use Cramer's Rule to solve each system of equations.

18. $a + b + 5c = 2$
$3a + b + 2c = 3$
$4a + 2b - c = -3$

19. $x + 3y - z = 5$
$2x + 5y - z = 12$
$x - 2y - 3z = -13$

20. $3c - 5d + 2e = 4$
$2c - 3d + 4c = -3$
$4c - 2d + 3e = 0$

21. $r - 4s - t = 6$
$2r - s + 3t = 0$
$3r - 2s + t = 4$

NAME ____ DATE ____________ PERIOD _____

4-6 Practice

Cramer's Rule

Use Cramer's Rule to solve each system of equations.

1. $2x + y = 0$
$3x + 2y = -2$

2. $5c + 9d = 19$
$2c - d = -20$

3. $2x + 3y = 5$
$3x - 2y = 1$

4. $20m - 3n = 28$
$2m + 3n = 16$

5. $x - 3y = 6$
$3x + y = -22$

6. $5x - 6y = -45$
$9x + 8y = 13$

7. $-2e + f = 4$
$-3e + 5f = -15$

8. $2x - y = -1$
$2x - 4y = 8$

9. $8a + 3b = 24$
$2a + b = 4$

10. $-3x + 15y = 45$
$-2x + 7y = 18$

11. $3u - 5v = 11$
$6u + 7v = -12$

12. $-6g + h = -10$
$-3g - 4h = 4$

13. $x - 3y = 8$
$x - 0.5y = 3$

14. $0.2x - 0.5y = -1$
$0.6x - 3y = -9$

15. $0.3d - 0.6g = 1.8$
$0.2d + 0.3g = 0.5$

16. GEOMETRY The two sides of an angle are contained in the lines whose equations are $x - \frac{4}{3}y = 6$ and $2x + y = 1$. Find the coordinates of the vertex of the angle.

17. GEOMETRY Two sides of a parallelogram are contained in the lines whose equations are $0.2x - 0.5y = 1$ and $0.02x - 0.3y = -0.9$. Find the coordinates of a vertex of the parallelogram.

Use Cramer's Rule to solve each system of equations.

18. $x + 3y + 3z = 4$
$-x + 2y + z = -1$
$4x + y - 2z = -1$

19. $-5a + b - 4c = 7$
$-3a + 2b - c = 0$
$2a + 3b - c = 17$

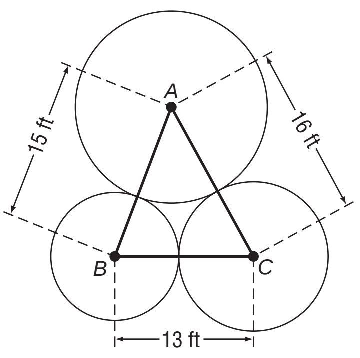
20. $2x + y - 3z = -5$
$5x + 2y - 2z = 8$
$3x - 3y + 5z = 17$

21. $2c + 3d - e = 17$
$4c + d + 5e = -9$
$c + 2d - e = 12$

22. $2j + k - 3m = -3$
$3j + 2k + 4m = 5$
$-4j - k + 2m = 4$

23. $3x - 2y + 5z = 3$
$2x + 2y - 4z = 3$
$-5x + 10y + 7z = -3$

24. LANDSCAPING A memorial garden being planted in front of a municipal library will contain three circular beds that are tangent to each other. A landscape architect has prepared a sketch of the design for the garden using CAD (computer-aided drafting) software, as shown at the right. The centers of the three circular beds are represented by points *A*, *B*, and *C*. The distance from *A* to *B* is 15 feet, the distance from *B* to *C* is 13 feet, and the distance from *A* to *C* is 16 feet. What is the radius of each of the circular beds?



4-6 Reading to Learn Mathematics

Cramer's Rule

Pre-Activity **How is Cramer's Rule used to solve systems of equations?**

Read the introduction to Lesson 4-6 at the top of page 189 in your textbook.

A triangle is bounded by the x-axis, the line $y = \frac{1}{2}x$, and the line $y = -2x + 10$. Write three systems of equations that you could use to find the three vertices of the triangle. (Do not actually find the vertices.)

Reading the Lesson

1. Suppose that you are asked to solve the following system of equations by Cramer's Rule.

$3x + 2y = 7$
$2x - 3y = 22$

Without actually evaluating any determinants, indicate which of the following ratios of determinants gives the correct value for x.

A. $\dfrac{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}{\begin{vmatrix} 7 & 2 \\ 22 & -3 \end{vmatrix}}$ **B.** $\dfrac{\begin{vmatrix} 7 & 2 \\ 22 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}$ **C.** $\dfrac{\begin{vmatrix} 3 & 7 \\ 2 & 22 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix}}$

2. In your textbook, the statements of Cramer's Rule for two variables and three variables specify that the determinant formed from the coefficients of the variables cannot be 0. If the determinant is zero, what do you know about the system and its solutions?

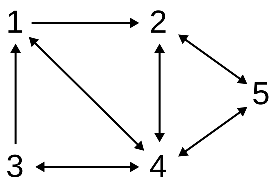
Helping You Remember

3. Some students have trouble remembering how to arrange the determinants that are used in solving a system of two linear equations by Cramer's Rule. What is a good way to remember this?

4-6 Enrichment

Communications Networks

The diagram at the right represents a communications network linking five computer remote stations. The arrows indicate the direction in which signals can be transmitted and received by each computer. We can generate a matrix to describe this network.



$$A = \underset{\text{computer } i}{\text{from}} \overset{\text{to computer } j}{\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}}$$

The entry in position a_{ij} represents the number of ways to send a message from computer i to computer j directly. Compare the entries of matrix A to the diagram to verify the entries. For example, there is one way to send a message from computer 3 to computer 4, so $A_{3,4} = 1$. A computer cannot send a message to itself, so $A_{1,1} = 0$.

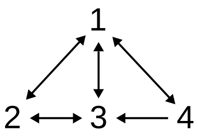
Matrix A is a communications network for direct communication. Suppose you want to send a message from one computer to another using exactly one other computer as a relay point. It can be shown that the entries of matrix A^2 represent the number of ways to send a message from one point to another by going through a third station. For example, a message may be sent from station 1 to station 5 by going through station 2 or station 4 on the way. Therefore, ${A^2}_{1,5} = 2$.

$$A^2 = \underset{\text{computer } i}{\text{from}} \overset{\text{to computer } j}{\begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 2 & 0 & 4 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}}$$

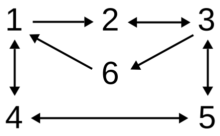
Again, compare the entries of matrix A^2 to the communications diagram to verify that the entries are correct. Matrix A^2 represents using exactly one relay.

For each network, find the matrices A and A^2. Then write the number of ways the messages can be sent for each matrix.

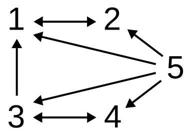
1.



2.



3.



NAME ______________________________ DATE ____________ PERIOD _____

4-7 Study Guide and Intervention

Identity and Inverse Matrices

Identity and Inverse Matrices The identity matrix for matrix multiplication is a square matrix with 1s for every element of the main diagonal and zeros elsewhere.

Identity Matrix for Multiplication	If A is an $n \times n$ matrix and I is the identity matrix, then $A \cdot I = A$ and $I \cdot A = A$.

If an $n \times n$ matrix A has an inverse A^{-1}, then $A \cdot A^{-1} = A^{-1} \cdot A = I$.

Example **Determine whether $X = \begin{bmatrix} 7 & 4 \\ 10 & 6 \end{bmatrix}$ and $Y = \begin{bmatrix} 3 & -2 \\ -5 & \frac{7}{2} \end{bmatrix}$ are inverse matrices.**

Find $X \cdot Y$.

$$X \cdot Y = \begin{bmatrix} 7 & 4 \\ 10 & 6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ -5 & \frac{7}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 21 - 20 & -14 + 14 \\ 30 - 30 & -20 + 21 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find $Y \cdot X$.

$$Y \cdot X = \begin{bmatrix} 3 & -2 \\ -5 & \frac{7}{2} \end{bmatrix} \cdot \begin{bmatrix} 7 & 4 \\ 10 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 21 - 20 & 12 - 12 \\ -35 + 35 & -20 + 21 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since $X \cdot Y = Y \cdot X = I$, X and Y are inverse matrices.

Exercises

Determine whether each pair of matrices are inverses.

1. $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$ and $\begin{bmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix}$

3. $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

4. $\begin{bmatrix} 8 & 11 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} -4 & 11 \\ 3 & -8 \end{bmatrix}$

5. $\begin{bmatrix} 4 & -1 \\ 5 & 3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$

6. $\begin{bmatrix} 5 & 2 \\ 11 & 4 \end{bmatrix}$ and $\begin{bmatrix} -2 & 1 \\ \frac{11}{2} & -\frac{5}{2} \end{bmatrix}$

7. $\begin{bmatrix} 4 & 2 \\ 6 & -2 \end{bmatrix}$ and $\begin{bmatrix} -\frac{1}{5} & -\frac{1}{10} \\ \frac{3}{10} & \frac{1}{10} \end{bmatrix}$

8. $\begin{bmatrix} 5 & 8 \\ 4 & 6 \end{bmatrix}$ and $\begin{bmatrix} -3 & 4 \\ 2 & -\frac{5}{2} \end{bmatrix}$

9. $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$ and $\begin{bmatrix} \frac{7}{2} & -\frac{3}{2} \\ 1 & -2 \end{bmatrix}$

10. $\begin{bmatrix} 3 & 2 \\ 4 & -6 \end{bmatrix}$ and $\begin{bmatrix} 3 & 2 \\ -4 & -3 \end{bmatrix}$

11. $\begin{bmatrix} 7 & 2 \\ 17 & 5 \end{bmatrix}$ and $\begin{bmatrix} 5 & -2 \\ -17 & 7 \end{bmatrix}$

12. $\begin{bmatrix} 4 & 3 \\ 7 & 5 \end{bmatrix}$ and $\begin{bmatrix} -5 & 3 \\ 7 & -4 \end{bmatrix}$

4-7 Study Guide and Intervention *(continued)*

Identity and Inverse Matrices

Find Inverse Matrices

Inverse of a 2 × 2 Matrix	The inverse of a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.

If $ad - bc = 0$, the matrix does not have an inverse.

Example **Find the inverse of $N = \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix}$.**

First find the value of the determinant.

$$\begin{vmatrix} 7 & 2 \\ 2 & 1 \end{vmatrix} = 7 - 4 = 3$$

Since the determinant does not equal 0, N^{-1} exists.

$$N^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix}$$

Check:

$$NN^{-1} = \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix} = \begin{bmatrix} \frac{7}{3} - \frac{4}{3} & -\frac{14}{3} + \frac{14}{3} \\ \frac{2}{3} - \frac{2}{3} & -\frac{4}{3} + \frac{7}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$N^{-1}N = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix} \cdot \begin{bmatrix} 7 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{7}{3} - \frac{4}{3} & \frac{2}{3} - \frac{2}{3} \\ -\frac{14}{3} + \frac{14}{3} & -\frac{4}{3} + \frac{7}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Exercises

Find the inverse of each matrix, if it exists.

1. $\begin{bmatrix} 24 & 12 \\ 8 & 4 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 40 & -10 \\ -20 & 30 \end{bmatrix}$

4. $\begin{bmatrix} 6 & 5 \\ 10 & 8 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$

6. $\begin{bmatrix} 8 & 2 \\ 10 & 4 \end{bmatrix}$

NAME ______________________________ DATE ____________ PERIOD _____

4-7 Skills Practice

Identity and Inverse Matrices

Lesson 4-7

Determine whether each pair of matrices are inverses.

1. $X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

2. $P = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}, Q = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

3. $M = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}, N = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$

4. $A = \begin{bmatrix} -2 & 5 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$

5. $V = \begin{bmatrix} 0 & 7 \\ -7 & 0 \end{bmatrix}, W = \begin{bmatrix} 0 & -\frac{1}{7} \\ \frac{1}{7} & 0 \end{bmatrix}$

6. $X = \begin{bmatrix} -1 & 4 \\ 1 & 2 \end{bmatrix}, Y = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$

7. $G = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}, H = \begin{bmatrix} \frac{2}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{4}{11} \end{bmatrix}$

8. $D = \begin{bmatrix} -4 & -4 \\ -4 & 4 \end{bmatrix}, E = \begin{bmatrix} -0.125 & -0.125 \\ -0.125 & -0.125 \end{bmatrix}$

Find the inverse of each matrix, if it exists.

9. $\begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

11. $\begin{bmatrix} 9 & 3 \\ 6 & 2 \end{bmatrix}$

12. $\begin{bmatrix} -2 & -4 \\ 6 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 1 & -1 \\ 3 & 3 \end{bmatrix}$

14. $\begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$

15. $\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$

16. $\begin{bmatrix} -4 & 5 \\ 1 & 2 \end{bmatrix}$

17. $\begin{bmatrix} 0 & -7 \\ -7 & 0 \end{bmatrix}$

18. $\begin{bmatrix} 10 & 8 \\ 5 & 4 \end{bmatrix}$

19. $\begin{bmatrix} 10 & 8 \\ 10 & -8 \end{bmatrix}$

20. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

NAME ____ DATE ____________ PERIOD _____

4-7 Practice

Identity and Inverse Matrices

Determine whether each pair of matrices are inverses.

1. $M = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, N = \begin{bmatrix} -2 & 1 \\ 3 & -2 \end{bmatrix}$

2. $X = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}, Y = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

3. $A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{5} & -\frac{1}{10} \\ \frac{2}{5} & \frac{3}{10} \end{bmatrix}$

4. $P = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix}, Q = \begin{bmatrix} \frac{3}{14} & \frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$

Determine whether each statement is *true* or *false*.

5. All square matrices have multiplicative inverses.

6. All square matrices have multiplicative identities.

Find the inverse of each matrix, if it exists.

7. $\begin{bmatrix} 4 & 5 \\ -4 & -3 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

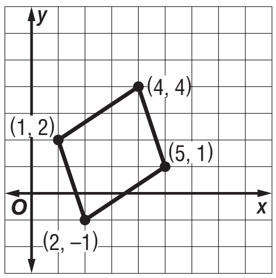
9. $\begin{bmatrix} -1 & 3 \\ 4 & -7 \end{bmatrix}$

10. $\begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix}$

11. $\begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$

12. $\begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix}$

GEOMETRY For Exercises 13–16, use the figure at the right.



13. Write the vertex matrix A for the rectangle.

14. Use matrix multiplication to find BA if $B = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$.

15. Graph the vertices of the transformed triangle on the previous graph. Describe the transformation.

16. Make a conjecture about what transformation B^{-1} describes on a coordinate plane.

17. CODES Use the alphabet table below and the inverse of coding matrix $C = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ to decode this message:

19 | 14 | 11 | 13 | 11 | 22 | 55 | 65 | 57 | 60 | 2 | 1 | 52 | 47 | 33 | 51 | 56 | 55.

CODE						
A 1	**B** 2	**C** 3	**D** 4	**E** 5	**F** 6	**G** 7
H 8	**I** 9	**J** 10	**K** 11	**L** 12	**M** 13	**N** 14
O 15	**P** 16	**Q** 17	**R** 18	**S** 19	**T** 20	**U** 21
V 22	**W** 23	**X** 24	**Y** 25	**Z** 26	**–** 0	

NAME ____ DATE ____________ PERIOD _____

4-7 Reading to Learn Mathematics

Identity and Inverse Matrices

Pre-Activity **How are inverse matrices used in cryptography?**

Read the introduction to Lesson 4-7 at the top of page 195 in your textbook.

Refer to the code table given in the introduction to this lesson. Suppose that you receive a message coded by this system as follows:

16 12 5 1 19 5   2 5   13 25   6 18 9 5 14 4.

Decode the message.

Lesson 4-7

Reading the Lesson

1. Indicate whether each of the following statements is *true* or *false*.

a. Every element of an identity matrix is 1.

b. There is a 3×2 identity matrix.

c. Two matrices are inverses of each other if their product is the identity matrix.

d. If M is a matrix, M^{-1} represents the reciprocal of M.

e. No 3×2 matrix has an inverse.

f. Every square matrix has an inverse.

g. If the two columns of a 2×2 matrix are identical, the matrix does not have an inverse.

2. Explain how to find the inverse of a 2×2 matrix. Do not use any special mathematical symbols in your explanation.

Helping You Remember

3. One way to remember something is to explain it to another person. Suppose that you are studying with a classmate who is having trouble remembering how to find the inverse of a 2×2 matrix. He remembers how to move elements and change signs in the matrix, but thinks that he should multiply by the determinant of the original matrix. How can you help him remember that he must multiply by the *reciprocal* of this determinant?

4-7 Enrichment

Permutation Matrices

A permutation matrix is a square matrix in which each row and each column has one entry that is 1. All the other entries are 0. Find the inverse of a permutation matrix interchanging the rows and columns. For example, row 1 is interchanged with column 1, row 2 is interchanged with column 2.

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad P^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

P is a 4×4 permutation matrix. P^{-1} is the inverse of P.

Solve each problem.

1. There is just one 2×2 permutation matrix that is not also an identity matrix. Write this matrix.

2. Find the inverse of the matrix you wrote in Exercise 1. What do you notice?

3. Show that the two matrices in Exercises 1 and 2 are inverses.

4. Write the inverse of this matrix.

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

5. Use B^{-1} from problem 4. Verify that B and B^{-1} are inverses.

6. Permutation matrices can be used to write and decipher codes. To see how this is done, use the message matrix M and matrix B from problem 4. Find matrix C so that C equals the product MB. Use the rules below.

0 times a letter = 0
1 times a letter = the same letter
0 plus a letter = the same letter

$$M = \begin{bmatrix} \text{S} & \text{H} & \text{E} \\ \text{S} & \text{A} & \text{W} \\ \text{H} & \text{I} & \text{M} \end{bmatrix}$$

7. Now find the product CB^{-1}. What do you notice?

4-8 Study Guide and Intervention

Using Matrices to Solve Systems of Equations

Write Matrix Equations A **matrix equation** for a system of equations consists of the product of the coefficient and variable matrices on the left and the constant matrix on the right of the equals sign.

Example **Write a matrix equation for each system of equations.**

a. $\mathbf{3x - 7y = 12}$
$\mathbf{x + 5y = -8}$

Determine the coefficient, variable, and constant matrices.

$$\begin{bmatrix} 3 & -7 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$$

b. $\mathbf{2x - y + 3z = -7}$
$\mathbf{x + 3y - 4z = 15}$
$\mathbf{7x + 2y + z = -28}$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -4 \\ 7 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 15 \\ -28 \end{bmatrix}$$

Exercises

Write a matrix equation for each system of equations.

1. $2x + y = 8$
$5x - 3y = -12$

2. $4x - 3y = 18$
$x + 2y = 12$

3. $7x - 2y = 15$
$3x + y = -10$

4. $4x - 6y = 20$
$3x + y + 8 = 0$

5. $5x + 2y = 18$
$x = -4y + 25$

6. $3x - y = 24$
$3y = 80 - 2x$

7. $2x + y + 7z = 12$
$5x - y + 3z = 15$
$x + 2y - 6z = 25$

8. $5x - y + 7z = 32$
$x + 3y - 2z = -18$
$2x + 4y - 3z = 12$

9. $4x - 3y - z = -100$
$2x + y - 3z = -64$
$5x + 3y - 2z = 8$

10. $x - 3y + 7z = 27$
$2x + y - 5z = 48$
$4x - 2y + 3z = 72$

11. $2x + 3y - 9z = -108$
$x + 5z = 40 + 2y$
$3x + 5y = 89 + 4z$

12. $z = 45 - 3x + 2y$
$2x + 3y - z = 60$
$x = 4y - 2z + 120$

Lesson 4-8

NAME ______________________________ DATE __________ PERIOD _____

4-8 Study Guide and Intervention *(continued)*

Using Matrices to Solve Systems of Equations

Solve Systems of Equations Use inverse matrices to solve systems of equations written as matrix equations.

Solving Matrix Equations	If $AX = B$, then $X = A^{-1}B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix.

Example **Solve** $\begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

In the matrix equation $A = \begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$.

Step 1 Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{20 - 12}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix} \text{ or } \frac{1}{8}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix}.$$

Step 2 Multiply each side of the matrix equation by the inverse matrix.

$$\frac{1}{8}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$ Multiply each side by A^{-1}.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 16 \\ -16 \end{bmatrix}$$ Multiply matrices.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$ Simplify.

The solution is (2, −2).

Exercises

Solve each matrix equation or system of equations by using inverse matrices.

1. $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 18 \end{bmatrix}$

2. $\begin{bmatrix} -4 & -8 \\ 6 & 12 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

4. $\begin{bmatrix} 2 & -3 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 6 \\ 5 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -15 \\ 6 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$

7. $4x - 2y = 22$
$6x + 4y = -2$

8. $2x - y = 2$
$x + 2y = 46$

9. $3x + 4y = 12$
$5x + 8y = -8$

10. $x + 3y = -5$
$2x + 7y = 8$

11. $5x + 4y = 5$
$9x - 8y = 0$

12. $3x - 2y = 5$
$x - 4y = 20$

NAME ______________________________ DATE ____________ PERIOD _____

4-8 Skills Practice

Using Matrices to Solve Systems of Equations

Write a matrix equation for each system of equations.

1. $x + y = 5$
$2x - y = 1$

2. $3a + 8b = 16$
$4a + 3b = 3$

3. $m + 3n = -3$
$4m + 3n = 6$

4. $2c + 3d = 6$
$3c - 4d = 7$

5. $r - s = 1$
$2r + 3s = 12$

6. $x + y = 5$
$3x + 2y = 10$

7. $6x - y + 2z = -4$
$-3x + 2y - z = 10$
$x + y + z = 3$

8. $a - b + c = 5$
$3a + 2b - c = 0$
$2a + 3b = 8$

Lesson 4-8

Solve each matrix equation or system of equations by using inverse matrices.

9. $\begin{bmatrix} 1 & 3 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} w \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ -1 \end{bmatrix}$

10. $\begin{bmatrix} 4 & 3 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$

11. $\begin{bmatrix} 5 & 8 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

12. $\begin{bmatrix} 7 & -3 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 15 \\ 23 \end{bmatrix}$

13. $\begin{bmatrix} 3 & 12 \\ 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 25 \\ 12 \end{bmatrix}$

14. $\begin{bmatrix} 5 & 6 \\ 12 & -6 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 15 \\ 2 \end{bmatrix}$

15. $p - 3q = 6$
$2p + 3q = -6$

16. $-x - 3y = 2$
$-4x - 5y = 1$

17. $2m + 2n = -8$
$6m + 4n = -18$

18. $-3a + b = -9$
$5a - 2b = 14$

NAME ____ DATE ____________ PERIOD _____

4-8 Practice

Using Matrices to Solve Systems of Equations

Write a matrix equation for each system of equations.

1. $-3x + 2y = 9$
$5x - 3y = -13$

2. $6x - 2y = -2$
$3x + 3y = 10$

3. $2a + b = 0$
$3a + 2b = -2$

4. $r + 5s = 10$
$2r - 3s = 7$

5. $3x - 2y + 5z = 3$
$x + y - 4z = 2$
$-2x + 2y + 7z = -5$

6. $2m + n - 3p = -5$
$5m + 2n - 2p = 8$
$3m - 3n + 5p = 17$

Solve each matrix equation or system of equations by using inverse matrices.

7. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} g \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

8. $\begin{bmatrix} -2 & 3 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 10 \end{bmatrix}$

9. $\begin{bmatrix} -1 & -3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 12 \\ -11 \end{bmatrix}$

10. $\begin{bmatrix} -5 & 3 \\ 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 16 \\ 34 \end{bmatrix}$

11. $\begin{bmatrix} -4 & 2 \\ 7 & 4 \end{bmatrix} \cdot \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 17 \\ -26 \end{bmatrix}$

12. $\begin{bmatrix} 8 & 3 \\ 12 & 6 \end{bmatrix} \cdot \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

13. $2x + 3y = 5$
$3x - 2y = 1$

14. $8d + 9f = 13$
$-6d + 5f = -45$

15. $5m + 9n = 19$
$2m - n = -20$

16. $-4j + 9k = -8$
$6j + 12k = -5$

17. AIRLINE TICKETS Last Monday at 7:30 A.M., an airline flew 89 passengers on a commuter flight from Boston to New York. Some of the passengers paid $120 for their tickets and the rest paid $230 for their tickets. The total cost of all of the tickets was $14,200. How many passengers bought $120 tickets? How many bought $230 tickets?

18. NUTRITION A single dose of a dietary supplement contains 0.2 gram of calcium and 0.2 gram of vitamin C. A single dose of a second dietary supplement contains 0.1 gram of calcium and 0.4 gram of vitamin C. If a person wants to take 0.6 gram of calcium and 1.2 grams of vitamin C, how many doses of each supplement should she take?

4-8 Reading to Learn Mathematics

Using Matrices to Solve Systems of Equations

Pre-Activity **How are inverse matrices used in population ecology?**

Read the introduction to Lesson 4-8 at the top of page 202 in your textbook.

Write a 2×2 matrix that summarizes the information given in the introduction about the food and territory requirements for the two species.

Reading the Lesson

1. a. Write a matrix equation for the following system of equations.

$3x + 5y = 10$
$2x - 4y = -7$

b. Explain how to use the matrix equation you wrote above to solve the system. Use as few mathematical symbols in your explanation as you can. Do not actually solve the system.

2. Write a system of equations that corresponds to the following matrix equation.

$$\begin{bmatrix} 3 & 2 & -4 \\ 2 & -1 & 0 \\ 0 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ -4 \end{bmatrix}$$

Helping You Remember

3. Some students have trouble remembering how to set up a matrix equation to solve a system of linear equations. What is an easy way to remember the order in which to write the three matrices that make up the equation?

NAME ____ DATE ____________ PERIOD _____

4-8 Enrichment

Properties of Matrices

Computing with matrices is different from computing with real numbers. Stated below are some properties of the real number system. Are these also true for matrices? In the problems on this page, you will investigate this question.

For all real numbers a and b, $ab = 0$ if and only if $a = 0$ or $b = 0$.

Multiplication is commutative. For all real numbers a and b, $ab = ba$.

Multiplication is associative. For all real numbers a, b, and c, $a(bc) = (ab)c$.

Use the matrices *A*, *B*, and *C* for the problems. Write whether each statement is true. Assume that a 2-by-2 matrix is the 0 matrix if and only if all of its elements are zero.

$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -3 \\ -1 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$

1. $AB = 0$

2. $AC = 0$

3. $BC = 0$

4. $AB = BA$

5. $AC = CA$

6. $BC = CB$

7. $A(BC) = (AB)C$

8. $B(CA) = (BC)A$

9. $B(AC) = (BA)C$

10. Write a statement summarizing your findings about the properties of matrix multiplication.